HOMEWORK 5
DIFFERENTIAL EQUATIONS
DUE 2013-09-05

Show your work!

(1) We saw in class that, if your bank account has 3% APY, compounded continuously, and if you make continuous withdrawals at a rate of \$100/month, then your balance B (in dollars) is governed by the differential equation

$$\frac{\mathrm{d}B}{\mathrm{d}t} = 0.03B - 1200,$$

where t is measured in years.

(a) Solve this differential equation. (HINT: What kind of equation is it?)

(b) If your initial balance is $\$100,000$, then how much money will you have after 1 year?

(c) If your initial balance is $\$10,000$, then how long will it take you to go broke? (HINT: What is the value of B when you go broke?)

(d) What is the minimum amount of money you need to invest in order never to go broke?

(2) Suppose that a 100 gal tank is initially *half* full of pure water. It is fed at a rate of 3 gal/min by a supply pipe dispensing sugar water at a concentration of 1/4 lb/gal. The sugar water is mixed continuously, and drained at a rate of 2 gal/min.

(a) What is the formula for the amount of *water* in the tank at time t?

(b) Set up a differential equation for the amount of *sugar* in the tank at time t.

(c) Solve the differential equation from (b). (HINT: What kind of equation is it?)

(d) How much sugar is in the tank when it overflows? (HINT: What is the initial condition? At what time t does the tank overflow?)

- **Four** book problems: #2.1.1, 8; #2.3.5, 9.