HOMEWORK 4
DIFFERENTIAL EQUATIONS
DUE 2013-09-03

Show your work!

(1) (a) Use separation of variables to find an implicit form of the solution to the initial-value problem

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{2\cos(2t)}{3+2y}, \quad y(0) = -1.$$

(b) Re-write your answer to (a) as a quadratic equation

$$ay^2 + by + c = 0.$$

(Your a, b, and c may depend on t.)

(c) Use the quadratic formula and your answer to (b) to find an explicit formula for y. (You will need to use the initial condition again to get rid of '±'.)

(2) Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} + q(t)y = g(t).$$

Suppose that $f(t)$ is an anti-derivative for $q(t)$ (that is, $f'(t) = q(t)$). Show that multiplying both sides of the original equation by the integrating factor $\mu(t) = e^{f(t)}$ gives an exact equation.

(3) (a) Re-write the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y + t$$

in the form

$$P(t)\frac{\mathrm{d}y}{\mathrm{d}t} + Q(t)y = G(t).$$

What are $P(t)$, $Q(t)$, and $G(t)$?

(b) Find an appropriate integrating factor μ so that, when you multiply both sides by μ, the left-hand side of the differential equation from (a) becomes $\frac{\mathrm{d}}{\mathrm{d}t}(\mu y)$.

(c) Solve the differential equation from (b).

(d) Solve the initial-value problem

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y + t, \quad y(0) = 0.$$

- **Three** book problems: #2.1.26 (just solve the equation) (1 problem), #2.2.11, 26 (2 problems).