HOMEWORK 21
DIFFERENTIAL EQUATIONS
DUE 10-26

Show your work! Write all final complex number (or complex function) answers in the form $a + bi$, where a and b are real numbers (or real functions).

(1) Compute the following products.
   (a) $(1 + i)^2$.
   (b) $(\sqrt{2} + i\sqrt{3})(\sqrt{2} - i\sqrt{3})$.
   (c) $(2 + i)(\sqrt{5} + 3i)$.
   (d) $(1 + i\sqrt{5})(\cos(\sqrt{5}t) + i\sin(\sqrt{5}t))$.

(2) Compute the following quotients.
   (a) $\dfrac{\cos(2t) + i\sin(2t)}{\sqrt{5} + 3i}$.
   (b) $\dfrac{2 + i}{\sin(t) + i\cos(t)}$.

(3) Consider the constant-coefficient, homogeneous, linear system with coefficient matrix $A = \begin{pmatrix} 0 & 2 \\ -3 & 2 \end{pmatrix}$.
   (a) In class, we found that $V = \begin{pmatrix} 2 \\ 1 + i\sqrt{5} \end{pmatrix}$ is an eigenvector of A with eigenvalue $\lambda = 1 + i\sqrt{5}$, so that

$$Y = \begin{pmatrix} 2e^{(1+i\sqrt{5})t} \\ (1 + i\sqrt{5})e^{(1+i\sqrt{5})t} \end{pmatrix}$$

   is a 'straight-line solution'. Use Euler's formula to write

$$Y = Y_{\mathrm{re}} + iY_{\mathrm{im}},$$

   where Y_{re} and Y_{im} involve real exponentials, sines, and cosines, but only real coefficients. (See p. 300.)
   (b) Verify that Y_{re} and Y_{im} are solutions to the system of differential equations.

(4) In class, we discussed the *conjugate* of a complex number, which is obtained by switching the sign of i. For example, the conjugate of $1 + i\sqrt{5}$ is $1 - i\sqrt{5}$.
   (a) Use the quadratic formula to show that, if a, b, and c are real numbers and the equation $a\lambda^2 + b\lambda + c = 0$ has complex solutions, then those solutions are complex conjugates.
   (b) Use Euler's formula to show that, if λ_1 and λ_2 are complex conjugates, then so are $e^{\lambda_1 t}$ and $e^{\lambda_2 t}$. (HINT: Write $\lambda_1 = a + ib$. What is λ_2?)
   (c) Find an eigenvector of the matrix A from #3 with eigenvalue $\lambda = 1 - i\sqrt{5}$. How is it related to the eigenvector in #3(a)?

- **One** book problem (with modifications): Find the complex form of *one* 'straight-line solution' to #3.4.10. Write it in the same form as for #3(a).