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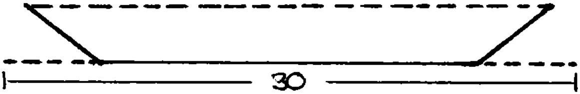
Trying to maximise area



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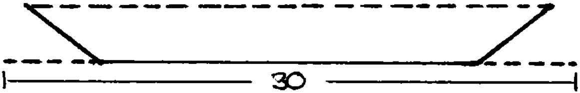
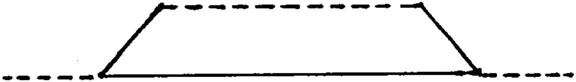


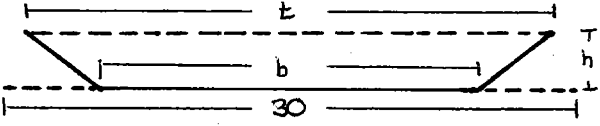
Why not like this?



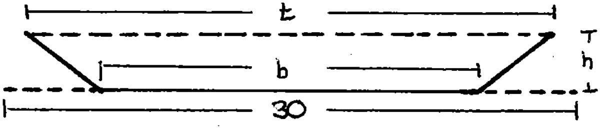
Area of a trapezoid:

$$A = \frac{1}{2}(t + b)h$$

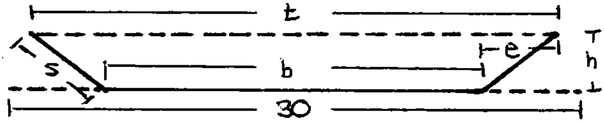
t and b are length of top and bottom, and h is the height (in inches)



Three input variables, t, b, h



Three input variables, t, b, h, plus two more, s and e:



$$b + 2s = 30 \Rightarrow b = 30 - 2s$$

By Pythagorean theorem,

$$s^2 = h^2 + e^2 \Rightarrow e = \sqrt{s^2 - h^2}$$

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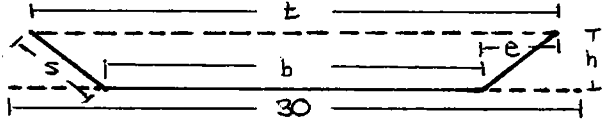

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and

$$t = b + 2e = 30 - 2s + 2\sqrt{s^2 - h^2}.$$

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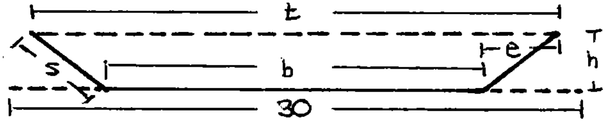

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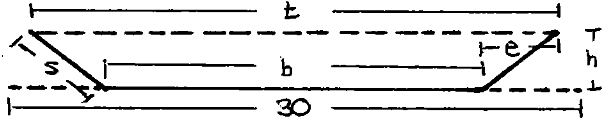

$$b = 30 - 2s \quad \text{and} \quad t = 30 - 2s + 2\sqrt{s^2 - h^2}$$

New formula:

$$A = \frac{1}{2}(t + b)h$$

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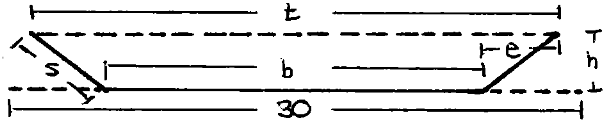

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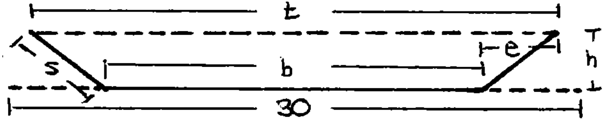

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Domain is (s, h) with $0 \leq s \leq 15$ and $0 \leq h \leq s$

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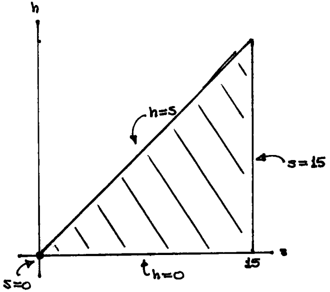
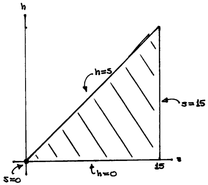
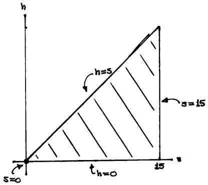




Interior of the domain is where all inequalities are strict:

$$0 < s < 15 \quad \text{and} \quad 0 < h < s.$$



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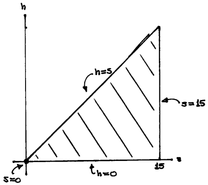
$$\nabla A = \left\langle \frac{\partial A}{\partial s}, \frac{\partial A}{\partial h} \right\rangle = \vec{0}$$

(or ∇A is undefined)



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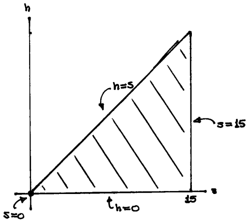
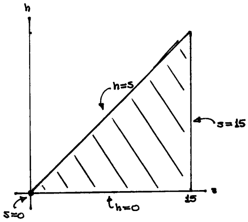
Be careful! There are solutions not in the interior of the domain



Boundary is where we have equality:

- $s = 0$ and $0 \leq h \leq s$



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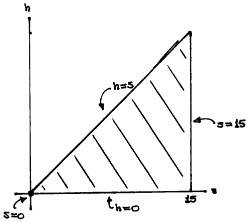

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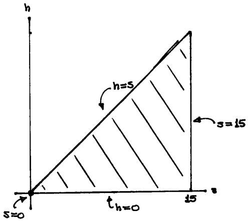

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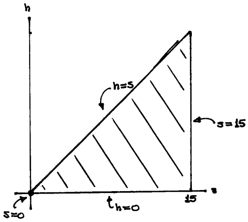

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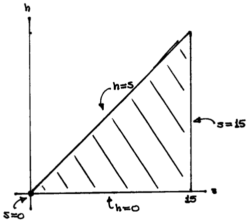

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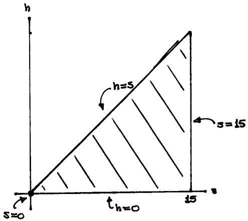

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- $s = 0$ and $h = 0$
- $s = 15$ and $0 \leq h \leq 15$
- $0 \leq s \leq 15$ and $h = 0$
- $0 \leq s \leq 15$ and $h = s$

Treat each case separately

When $s = 0$, the only possibility is $h = 0$, so add $(0, 0)$ to the list

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Endpoints $(s, h) = (15, 0)$ and $(s, h) = (15, 15)$

List so far:

- Interior critical point

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- ...

Plug all points (s, h) into A:

s	h	A
0	0	0
15	0	0
15	15	0
$\vdots$	$\vdots$	$\vdots$

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s	h	A
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The biggest are the absolute maxima

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s	h	A
0	0	0
15	0	0
15	15	0
$\vdots$	$\vdots$	$\vdots$

The biggest are the absolute maxima; no need for derivative tests