

HOMEWORK 12
CALCULUS III
DUE 02-28

(1) Consider the function

$$f(x, y) = \frac{x}{x^2 + y^2}.$$

- (a) The equation of the level curve at height $z = c$ can be re-written as the equation of a circle. In terms of c , what are its centre and radius?
- (b) According to your work in (a), the point $(0, 0)$ lies on *all* level curves; but, in class, we said that a point could only lie on *one* level curve. Explain this apparent contradiction.

(2) Consider the function

$$f(x, y) = \frac{x^2 y}{x^4 + y^2}.$$

- (a) Show that, as (x, y) approaches $(0, 0)$ along any line through the origin, $f(x, y)$ approaches 0. (HINT: What is the equation of a line through the origin? You will need to consider two different cases.)
 - (b) Find a path through the origin such that, as (x, y) approaches $(0, 0)$ along that path, $f(x, y)$ does *not* approach 0. (HINT: Try a parabola!)
- **Eleven** book problems: #13.1.37, 45–48, 53, 83, 86 (8 problems); #13.2.25, 28, 35 (3 problems).