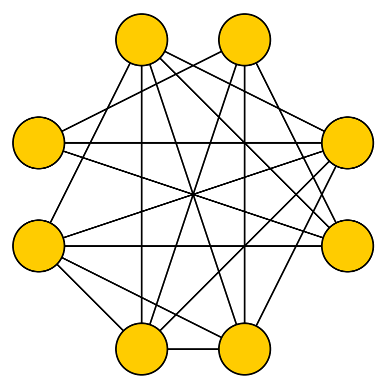
HOMEWORK 21
DISCRETE MATHEMATICS II
DUE 04-29

(1) (a) Give an example of a graph that satisfies the conditions of Ore's theorem (Theorem 10.5.4), and is therefore Hamiltonian.
   (b) Give an example of a graph that is Hamiltonian, but does not satisfy the conditions of Ore's theorem.
   (c) Give an example of a graph with more than 3 vertices that is not Hamiltonian. Show that it does not satisfy the conditions of Ore's theorem.

(2) The following is an algorithmic 'proof' of Ore's theorem (Theorem 10.5.4) for a graph G:
   (A) arrange the vertices of G along the perimeter of a circle. Say that v and w are CC if they are next to each other in the circle, and w is clockwise from v.
   (B) if the vertices are arranged so that any two CC vertices are adjacent in G, then stop. Otherwise, go to step (C).
   (C) if v and w are CC vertices that are not adjacent in G, then find two more CC vertices x and y such that v is adjacent to x and w is adjacent to y in G.
   (D) reverse the clockwise arc of the circle from w to x, and return to step (B).

   (See Ore's theorem on Wikipedia.) Use this algorithm to find a Hamiltonian circuit in



   Show the state of the graph after each time you perform step (D). (You must use the algorithm, even if you can find a circuit by trial and error.)

(3) Show that there is a vertex of the Petersen graph (see p. 706) such that deleting that vertex, and all edges incident with it, leaves a Hamiltonian graph. (This is the second half of #10.5.46.)

- **Three** book problems: #10.5.34, 37, 44(a–c).