

**HOMEWORK 20**  
**DISCRETE MATHEMATICS II**  
**DUE 04-15 (NOT 04-17)**

- (1) Suppose that you pick some simple path in a graph  $G$  that is not a cycle.
    - (a) Show that, if every vertex of  $G$  is even, then it is possible to extend the path to a longer one that is still simple. (HINT: First draw a picture to see what this means. Be sure *not* to use Euler's theorem (Theorem 10.5.1), since this problem is part of the proof of that theorem!)
    - (b) Give an example to show that (a) can fail if  $G$  has one or more vertices of odd degree.
  - (2) Suppose that you pick some path in a graph  $G$  that does *not* include all edges of  $G$ , and delete from  $G$  all edges occurring in that path.
    - (a) Show that, if the original graph  $G$  was connected, then there is some vertex that is part of the path and that is still incident with one or more (undeleted) edges.
    - (b) Give an example to show that (a) can fail if  $G$  is *not* connected.
- **Four** book problems: #10.5.1, 14, 18, 26. The graphs  $C_n$ ,  $W_n$ , and  $Q_n$  referenced in #10.5.26 are defined in Examples 10.2.6–8 on p. 655.