HOMEWORK 13
DISCRETE MATHEMATICS II
DUE 03-18; UPDATED 03-05

In class, I erroneously stated that $\varphi = \frac{\sqrt{5}-1}{2}$ and $\overline{\varphi} = -\left(\frac{\sqrt{5}+1}{2}\right)$. The correct values are the ones in #1. Please be sure to use them.

(1) Consider the polynomial $x^2 - x - 1$. Call its positive root φ and its negative root $-\overline{\varphi}$.
  (a) Explain informally why $x^2 - x - 1 = (x - \varphi)(x + \overline{\varphi})$.
  (b) Without using the quadratic formula or any numeric approximation, show that

$$\varphi \cdot \overline{\varphi} = 1 \quad \text{and} \quad \cancel{\varphi + \overline{\varphi} = 1} \quad \varphi = 1 + \overline{\varphi}$$

  (Hint: Use (a). These properties are why we call φ the *golden ratio*.)
  (c) ~~Without using the quadratic formula or any numeric approximation, show that $1/\varphi = \varphi/\overline{\varphi}$. (This property is why we call φ the *golden ratio*. Hint: Use (a, b).)~~

(2) Use the values of φ and $\overline{\varphi}$ from #1 (not the wrong values from class).
  (a) Show that $1 - x - x^2 = (1 + \overline{\varphi}x)(1 - \varphi x)$. (Hint: Use #1.)
  (b) Find constants A and B so that

$$\frac{x}{1 - x - x^2} = \frac{A}{1 + \overline{\varphi}x} + \frac{B}{1 - \varphi x}.$$

  (Hint: If necessary, review the technique of *partial fractions* from Calculus II. It may also help to use #1(b).)

- **Three** book problems: #8.4.16, 27, 35. For #8.4.16, you **must** use the techniques from class to get a numeric answer. For #8.4.35, just give the generating function for the sequence.