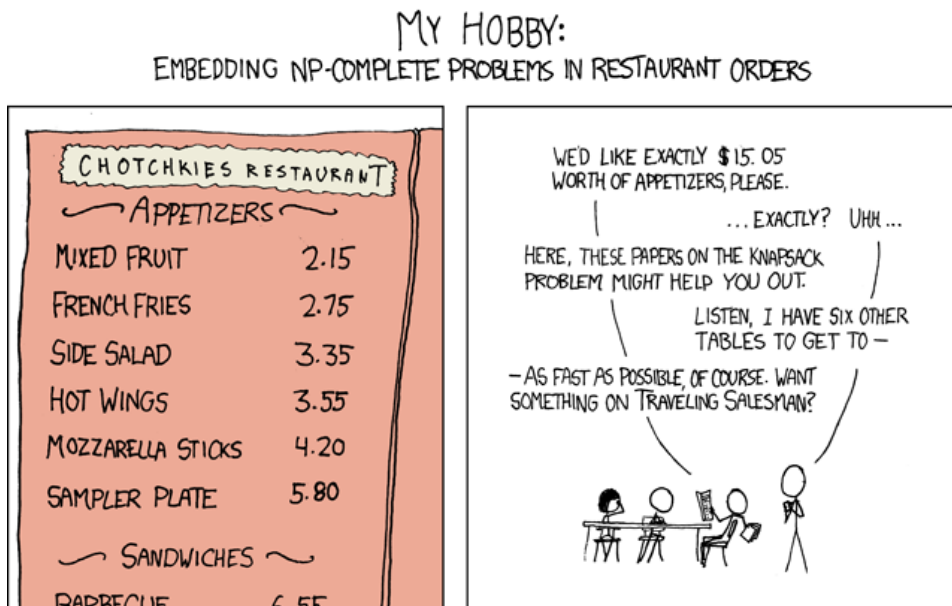


HOMEWORK 11
DISCRETE MATHEMATICS II
DUE 03-04

- (1) (a) Describe how to solve the Towers of Hanoi problem for 3 discs.
 (b) Describe how to solve the Towers of Hanoi problem for 4 discs.
 (2) Consider the following XKCD comic:



(<http://xkcd.com/287>). Let p_n be the price of the n th menu item, in cents. Thus, for example, $p_1 = 215$ and $p_2 = 275$. Let $a(n, p)$ be the number of ways of ordering from the first n items on the menu so that the total price is exactly p cents.

(a) We make the following slightly unusual choice of initial conditions:

- $a(0, 0) = 1$,
- $a(0, p) = 0$ for all $p \neq 0$, and
- $a(n, p) = 0$ for all $p < 0$.

Explain why these are sensible.

- (b) Fill out a table of the non-0 values of $a(1, p)$ and $a(2, p)$ with $p \leq 1505$. Explain. Note that you may order more than one of each item.
- (c) Write down a recurrence for $a(n, p)$ in terms of $a(n - 1, q)$, where p and q do not have to be the same. (HINT: Your formula will be a sum of terms, depending on how many of the n th item you order. It will involve the price p_n .)

- (3) Show that the generating function of the finite sequence $(1, 1, \dots, 1)$ is $\frac{x^m - 1}{x - 1}$ for some m . How is m related to the number of terms in the sequence?

- **Two** book problems: #8.1.38, #8.4.30.