HOMEWORK 10
DISCRETE MATHEMATICS II
DUE 02-27

(1) In class, we considered the number of ways of bracketing a product of n symbols. We count a single symbol like x as already bracketed, but insist that a pair of symbols appear with a bracket around them, like (xy). We wrote a_n for the number of possible bracketings, and discovered the recurrence

$$a_n = \sum_{i=1}^{n-1} a_i a_{n-i}, \quad a_1 = 1. \tag{$*$}$$

   (a) Explain why only one initial condition is needed, even though the sum in $(*)$ involves $n-1$ terms.
   (b) Use this recurrence to compute a_2, a_3, a_4, and a_5. Write out your work, and be sure that it actually uses $(*)$.
   (c) List all possible bracketings of $x_1x_2x_3x_4x_5$. How do you know that you have them all?

(2) In Example 8.1.5 in the text, the author finds a recurrence for the number of ways of bracketing a product of $n+1$ symbols. He writes C_n for the number of such bracketings. (Note that the subscript is n, but the number of symbols is $n+1$.) Explain how to use #1$(*)$ to produce a recurrence relation for C_n. (You must use #1$(*)$; it is not enough just to copy the recurrence from the text.)

(3) Let a_n be the number of paths from $(0,0)$ to (n,n) that never go above the line $y = x$, where a path is a series of steps, each of which is a move one unit to the right or one unit upwards. (Compare to #6.4.33.)
   (a) Find a recurrence relation for a_n. (HINT: Suppose that the first point where you hit the line $y = x$ (after $(0,0)$) is (i,i).)
   (b) What is the initial condition?
   (c) What can you say about the numbers a_n using only the answers to (a) and (b), *without* trying to solve the recurrence?

- **Three** book problems: #8.1.13, 24, 26. For #8.1.24, as in #8.1.7, it may help to look at Example 8.1.4.