## HOMEWORK 9

## DISCRETE MATHEMATICS II DUE 02-27 (NOT 02-25)

(1) In class, we considered the problem of counting the bit strings of length $n$ that do not contain two consecutive 0 's. We wrote $z_{n}$ for the number of such bit strings that end in 0 , $o_{n}$ for the number that end in 1 , and $a_{n}$ for the total number.
(a) What is the relationship among $a_{n}, z_{n}$, and $o_{n}$ ? Explain. (Pay attention to subscripts!)
(b) What is the relationship between $o_{n}$ and $a_{n-1}$ ? Explain. (Pay attention to subscripts!)
(c) Why did we choose to relate $o_{n}$, instead of $z_{n}$, to $a_{n-1}$ ?
(d) We found the equation

$$
a_{n}=z_{n-1}+2 o_{n-1}, \quad a_{1}=2, \quad z_{1}=1, \quad o_{1}=1 .
$$

Explain this equation in words that could be understood by a non-mathematician.
(e) Use your answers to (a) and (b), and the equation in (d), to write down a recurrence relation involving only $a$ 's. Be sure to include the initial condition.

- Four book problems: $\# 8.1 .3,4,7,11$. For $\# 8.1 .7$, you will probably also want to count the number of strings not satisfying the condition. See Example 8.1.4.

