HOMEWORK 22
DISCRETE MATHEMATICS I
DUE 04-30 (NOT 05-02)

(1) For each of the following statements, prove it or give a counterexample. These are *not* consequences of Theorem 3.8.
   (a) If $n \equiv 1 \pmod 2$, then $n^2 \equiv 1 \pmod 4$.
   (b) If $n \equiv 1 \pmod 3$, then $n^2 \equiv 1 \pmod 6$.
   (c) If $n \equiv 1 \pmod 3$, then $n^2 \equiv 1 \pmod 9$.

(2) (a) Prove that the 3-digit number abc is congruent to $a - b + c$ modulo 11.
   (b) Prove that the 3-digit number abc is divisible by 11 if and only if $a - b + c$ is divisible by 11. (HINT: Don't work too hard; once you've got (a), this is a 'one-liner'.)

(3) Let a and n be integers such that $\gcd(a, n) = 1$. Suppose that x and y are inverses of a modulo n.
   (a) Prove that $a(x - y) \equiv 0 \pmod n$.
   (b) Prove that $x - y \equiv 0 \pmod n$. (HINT: Be careful about cancellation (see #3.3.62). What allows you to get rid of the a?)
   (c) Prove that $x \equiv y \pmod n$. (HINT: Once you've got (b), this is a one-liner.)

- **Five** book problems: #3.3.18, 22, 30, 31, 62