HOMEWORK 21
DISCRETE MATHEMATICS I
DUE 04-30

(1) The definition of congruence on p. 85 of the notes assumes that the modulus is a positive integer, but it doesn't have to do so. Namely, we can make the following definition:

> For *any* integer n, two integers a and b are said to be congruent modulo n if and only if $n \mid (b-a)$. We denote this by $a \equiv b \pmod{n}$.

   (a) Let a be an integer. Describe $\{b \in \mathbb{Z} \,|\, a \equiv b \pmod{1}\}$.
   (b) Let a be an integer. Describe $\{b \in \mathbb{Z} \,|\, a \equiv b \pmod{0}\}$.

(2) For each of the following statements, prove it or give a counterexample.
   (a) If $ab \equiv 0 \pmod{2}$, then $a \equiv 0 \pmod{2}$ or $b \equiv 0 \pmod{2}$.
   (b) If $ab \equiv 0 \pmod{3}$, then $a \equiv 0 \pmod{3}$ or $b \equiv 0 \pmod{3}$.
   (c) If $ab \equiv 0 \pmod{4}$, then $a \equiv 0 \pmod{4}$ or $b \equiv 0 \pmod{4}$.

- **Six** book problems: #3.3.13, 14, 15, 58, 60, 61.