## HOMEWORK 20

## DISCRETE MATHEMATICS I

DUE 04-16 (NOT 04-18)
(1) Suppose that using the Euclidean algorithm on the numbers $a$ and $b$ gives a table that looks like:

| $Q$ | $R$ | $S$ | $T$ |
| :---: | :---: | :---: | :---: |
|  | $a$ | 1 | 0 |
|  | $b$ | 0 | 1 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $q_{n-2}$ | $r_{n-2}$ | $s_{n-2}$ | $t_{n-2}$ |
| $q_{n-1}$ | $r_{n-1}$ | $s_{n-1}$ | $t_{n-1}$ |
| $q_{n}$ | $r_{n}$ | $s_{n}$ | $t_{n}$ |

(a) Write formulæ for $r_{n}, s_{n}$, and $t_{n}$ in terms of $r_{n-2}$ and $r_{n-1}, s_{n-2}$ and $s_{n-1}, t_{n-2}$ and $t_{n-1}$, and $q_{n}$.
(b) Suppose that you already know that

$$
r_{n-2}=a s_{n-2}+b t_{n-2}
$$

and

$$
r_{n-1}=a s_{n-1}+b t_{n-1}
$$

Prove that

$$
r_{n}=a s_{n}+b t_{n} .
$$

- Eight book problems: \#3.1.21, 40, 42; \#3.2.5, 15, 24, 26, 27.

