

**HOMEWORK 16**  
**DISCRETE MATHEMATICS I**  
**DUE 04-04**

- (1) (a) Write down a formula for the ‘exclusive or’ set

$$A \Delta B = \{x \in U \mid (x \in A) \oplus (x \in B)\}$$

in terms of the existing set operations (union, intersection, complement, and relative difference). This new set is called the *symmetric difference* of  $A$  and  $B$ .

- (b) Compute the symmetric difference of  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{2, 4, 6, 8, 10\}$ .
- (2) (a) Find formulas for  $|A \times B|$  and  $|P(X)|$  in terms of  $|A|$ ,  $|B|$ , and  $|X|$ . (HINT: Try some examples first! You don’t have to *prove* the formula for  $|A \times B|$ .)
- (b) Use induction on  $n = |X|$  to prove that your formula for  $|P(X)|$  is correct.
- (c) Explain why  $P(X)$  is sometimes also denoted by  $2^X$ .
- (3) Explain the following sentence from class:

“ $\{3\} \in \{4, \{3\}\}$  but  $\{3\} \not\subseteq \{4, \{3\}\}$ , whereas  $\{4\} \subseteq \{4, \{3\}\}$  but  $\{4\} \notin \{4, \{3\}\}$ .”

- **Five** book problems: #2.1.17, 18, 19; #2.2.31, 32.