HOMEWORK 13
DISCRETE MATHEMATICS I
DUE 03-19

(1) This problem originally appeared on Homework #11. If you already submitted an answer, and you are confident that it is correct, then you do not need to re-submit it.

Use the following axioms of the integers:

AA: For all integers a, b, and c, we have that $(a+b)+c = a+(b+c)$.

AC: For all integers a and b, we have that $a+b = b+a$.

MI: There is an integer 1 so that, for all integers a, we have that $a \cdot 1 = a$ and $1 \cdot a = a$.

D: For all integers a, b, and c, we have that $ab + ac = a(b+c)$.

and the definition of 2 as $1+1$ to prove that, for all integers a and b, we have that

$$(2a+1)+(2b+1) = 2[(a+b)+1].$$

(Hint: Your proof can be just a string of equalities, but each step must be justified. Don't be tempted to skip steps! For me, this took nine steps.)

(2) Prove that every integer is even or odd. (Hint: You may use the facts that every integer is positive, negative, or 0, and that every *positive* integer is even or odd.)

(3) Prove that the sum of the first n odd integers is n^2.

- **Four** book problems: #5.1.1, 16, 17, 21. For #5.1.16, you may need the following definition (p. 70): an integer n is divisible by 4 if and only if it can be written in the form $n = 4q$ for some integer q. Hint: If you are proving a statement for all $n \geq 0$ (rather than just all positive integers n), what should your base case be?