HOMEWORK 11
DISCRETE MATHEMATICS I
LAST UPDATED 03-05
DUE 03-05

You may need the following fact about integers: "All integers are even or odd." This can be proven, but, for now, you should just assume it.

(1) ~~Prove that an integer that is not even is odd.~~

(1) Prove that any integer whose square is even is itself even.

(2) Use the following axioms of the integers:

AA: For all integers a, b, and c, we have that $(a + b) + c = a + (b + c)$.
AC: For all integers a and b, we have that ~~$ab = ba$~~ $a + b = b + a$.
MI: There is an integer 1 so that, for all integers a, we have that $a \cdot 1 = a$ and $1 \cdot a = a$.
D: For all integers a, b, and c, we have that $ab + ac = a(b + c)$.

(a) Axiom **MI** gives a definition of the integer 1. Give a definition of the integer 2. (Don't overthink it!)

(b) Use *only* the axioms (not just 'by algebra') to prove that, for all integers a and b, we have that

$$(2a + 1) + (2b + 1) = 2[(a + b) + 1].$$

(HINT: Your proof can be just a string of equalities, but each step must be justified. Don't be tempted to skip steps! For me, this took nine steps.)

(3) (a) Give a definition of the absolute value $|x|$ of a real number x.

(b) Explain why a statement involving the absolute value will probably need to be proved by cases.

(c) Book problem ~~#1.5.14~~ #1.5.12.

- **Five** book problems: #1.5.3, 5, 9, 11, 34.