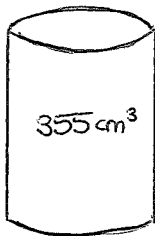


What is the cheapest Coke can that holds 355 cm^3 of soda?

Cheapest . . . : Minimum cost

Cost is proportional to surface area

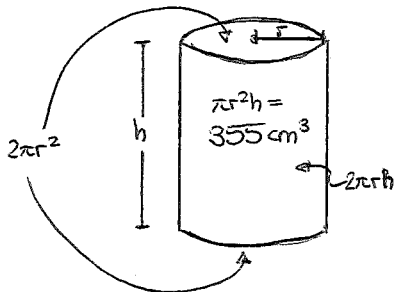
Coke can is a cylinder (approximately)



$$\text{Surface area: } SA = 2 \cdot \underbrace{\pi r^2}_{\text{base area}} + \underbrace{2\pi r}_{\text{base perim.}} \cdot h$$

$$\text{Volume: } V = \pi r^2 h = 355 \text{ (in cubic centimetres)}$$

r , h are radius, height of can (in centimetres)



Two independent variables, r and h

$$355 = \pi r^2 h \Rightarrow h = \frac{355}{\pi r^2} \text{ or } r = \sqrt{\frac{355}{\pi h}}$$

$$SA = 2\pi r^2 + 2\pi r \frac{355}{\pi r^2} = 2\pi r^2 + \frac{710}{r}$$

r is a length, so $r \geq 0$

$r = 0$ is impossible (would give $V = 0$)

r can be very big (if the can is short)



Domain is $(0, \infty)$

$$SA = 2\pi r^2 + 2\pi r \frac{355}{\pi r^2} = 2\pi r^2 + \frac{710}{r}$$
$$\frac{d}{dr}SA = 4\pi r - \frac{710}{r^2}$$

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Undefined at $r = 0$ (not in domain)

Zero when

$$4\pi r = \frac{710}{r^2}$$

$$4\pi r^3 = 710$$

$$r = \sqrt[3]{\frac{355}{2\pi}}$$

r	SA
0	∞
$\sqrt[3]{\frac{355}{2\pi}}$	277.5
∞	∞

So optimum dimensions are $r = \sqrt[3]{\frac{355}{2\pi}} \approx 3.837$,

$$h = \frac{355}{\pi r^2} = 2\sqrt[3]{\frac{355}{2\pi}} \approx 7.674 \text{ (in centimetres)}$$

Actual dimensions: $r \approx 3.2$ cm, $h \approx 11$ cm