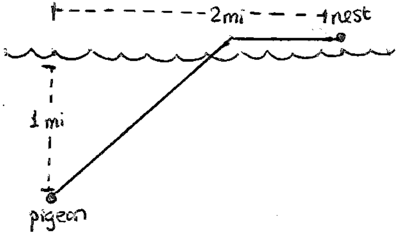
If it takes a pigeon only 75% as much energy to fly over land as over water, what path will it take when flying from a point 1 mi from shore to its nest which is 2 mi down the shoreline?

Pigeon wants to minimise energy

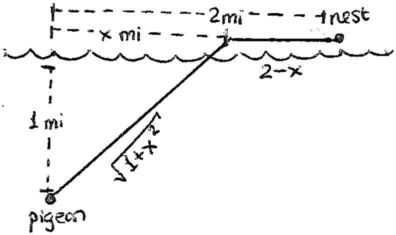
Best approach: fly straight to a point on the shore, then fly straight along the shoreline

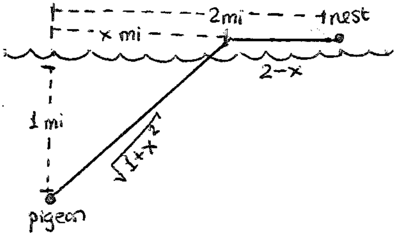


By Pythagorean theorem, distance flown over water is $\sqrt{1+x^2}$

Distance flown on land is $2 - x$

x is horizontal distance flown before reaching shoreline (in miles)





Total energy spent is

$$E_{\text{tot}} = E_{\text{water}} + E_{\text{land}} = k_{\text{water}} \cdot \sqrt{1 + x^2} + k_{\text{land}} \cdot (2 - x)$$

Given $k_{\text{land}} = 0.75 \cdot k_{\text{water}}$

Only one independent variable x; nothing to do!

$x = 0$: fly straight to shore

$x < 0$: start flying *away* from nest

$x = 2$: fly straight to nest

$x > 2$: fly *past* nest

Domain is $[0, 2]$

$$E_{\text{tot}} = k_{\text{water}} \cdot \sqrt{1 + x^2} + k_{\text{land}} \cdot (2 - x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x} E_{\text{tot}} = k_{\text{water}} \cdot \frac{x}{\sqrt{1 + x^2}} - k_{\text{land}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} E_{\text{tot}} = k_{\text{water}} \cdot \frac{x}{\sqrt{1+x^2}} - k_{\text{land}}$$

Denominator is never 0; always defined

$$\frac{\mathrm{d}}{\mathrm{d}x} E_{\mathrm{tot}} = k_{\mathrm{water}} \cdot \frac{x}{\sqrt{1+x^2}} - k_{\mathrm{land}}$$

$$\begin{aligned}
k_{\mathrm{water}} \cdot \frac{x}{\sqrt{1+x^2}} &= k_{\mathrm{land}} \\
k_{\mathrm{water}} \cdot x &= k_{\mathrm{land}} \cdot \sqrt{1+x^2} \\
k_{\mathrm{water}}^2 \cdot x^2 &= k_{\mathrm{land}}^2 (1+x^2) \\
(k_{\mathrm{water}}^2 - k_{\mathrm{land}}^2) x^2 &= k_{\mathrm{land}}^2 \\
x &= \pm \sqrt{\frac{k_{\mathrm{land}}^2}{k_{\mathrm{water}}^2 - k_{\mathrm{land}}^2}}
\end{aligned}$$

$$x = \pm\sqrt{\frac{k_{\text{land}}^2}{k_{\text{water}}^2 - k_{\text{land}}^2}}$$

Negative numbers are not in domain

Use $k_{\text{land}} = 0.75 \cdot k_{\text{water}}$, and cancel k_{water}^2 in numerator and denominator of expression for x:

$$x = \sqrt{\frac{0.75^2 \cdot k_{\text{water}}^2}{(1 - 0.75^2)k_{\text{water}}^2}} \approx \sqrt{1.286} \approx 1.134.$$

x	E_{tot}
0	$k_{water} + 2k_{land}$
1.134	$1.512 \cdot k_{water} + 0.8661 \cdot k_{land}$
2	$2.236 \cdot k_{water}$

Use $k_{land} = 0.75 \cdot k_{water}$:

x	E_{tot}
0	$3.5 \cdot k_{water}$
1.134	$2.161 \cdot k_{water}$
2	$2.236 \cdot k_{water}$

Best path is to fly straight to a point 1.134 mi along the shoreline, then fly the rest of the way along the shoreline