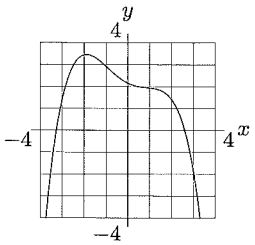
WEEKLY 8
APPLIED CALCULUS

Show your work!

This homework is due **Tuesday**, October 16, by the end of office hours (1 PM). You may also hand it in **in class** on Monday, October 15.

(1) There is no need to give formulas for the functions below.
   (a) Whether a *function* is increasing or decreasing has no relation to whether its *slope* is increasing or decreasing. There are four possible combinations of "function increasing vs. decreasing" and "slope increasing vs. decreasing". Sketch a graph for each, and label your graphs.
   (b) Sketch a graph of a function that is always decreasing, but that changes concavity.
   (c) Can a function whose *slope* is always increasing change from decreasing to increasing? Can such a function change from increasing to decreasing? In each case, sketch a graph if possible, and otherwise explain why it is not possible.

(2) Sketch (on separate axes) the derivative and second derivative of the function $y = f(x)$ graphed below.



(3) A small aircraft starts its descent to its destination airport from an altitude of 1 mile, 4 miles west of the runway.
   (a) The glide path of the airplane is modelled by $y = ax^3 + bx^2 + cx + d$, where x is the distance from the runway (in miles) and y is the height of the plane (in miles). What choice of coefficients a, b, c, and d will ensure a smooth landing? (HINT: We are told the value of $f(4)$, and the value of $f(0)$ is determined by the fact that the plane should end up on the ground. What additional constraints do we need for a smooth landing?)
   (b) When will the plane be descending at the greatest rate?

(4) Two people, Alex and Beth, are in an on-off relationship. The more Beth likes Alex, the more strongly he starts to dislike her. The more Alex likes Beth, the more strongly she starts to like him. This is a dysfunctional relationship!

If $a(t)$ is the amount that Alex likes Beth at time t, and $b(t)$ is the amount that Beth likes Alex at time t, then $a'(t) = -b(t)$ and $b'(t) = a(t)$.

Make a sketch of $a(t)$ and $b(t)$ on the same set of axes, if Alex is initially completely committed to Beth ($a(0) = 1$) and Beth is initially completely indifferent to Alex ($b(0) = 0$). Explain how you produced your graphs, **without** referring to any formula.

- **One** book problem: #12.4.20. Be sure to identify horizontal and vertical asymptotes (using a calculator if necessary). **Show your work**, *without* using your calculator to graph.