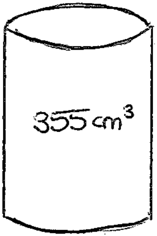
What is the cheapest Coke can that holds 355 cm^3 of soda?

Cheapest . . . : Minimum cost

Cost is proportional to surface area

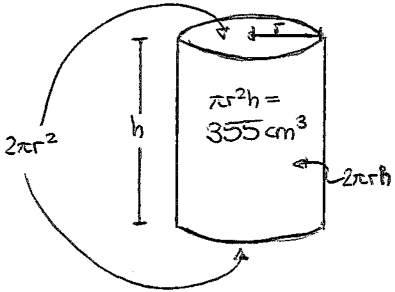
Coke can is a cylinder (approximately)



Surface area: $\mathrm{SA} = 2 \cdot \underbrace{\pi r^2}_{\substack{\text{base}\\\text{area}}} + \underbrace{2\pi r}_{\substack{\text{base}\\\text{perim.}}} \cdot h$

Volume: $\mathrm{V} = \pi r^2 h = 355$ (in cubic centimetres)

r, h are radius, height of can (in centimetres)



Two independent variables, r and h

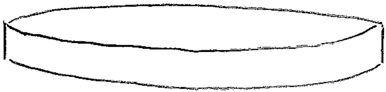
$$355 = \pi r^2 h \Rightarrow h = \frac{355}{\pi r^2} \text{ or } r = \sqrt{\frac{355}{\pi h}}$$

$$\text{SA} = 2\pi r^2 + 2\pi r \frac{355}{\pi r^2} = 2\pi r^2 + \frac{710}{r}$$

r is a length, so $r \geq 0$

$r = 0$ is impossible (would give $V = 0$)

r can be very big (if the can is short)



Domain is $(0, \infty)$

$$\mathrm{SA} = 2\pi r^2 + 2\pi r \frac{355}{\pi r^2} = 2\pi r^2 + \frac{710}{r}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\mathrm{SA} = 4\pi r - \frac{710}{r^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\mathrm{SA} = 4\pi r - \frac{710}{r^2}$$

Undefined at $r = 0$ (not in domain)

Zero when

$$4\pi r = \frac{710}{r^2}$$

$$4\pi r^3 = 710$$

$$r = \sqrt[3]{\frac{355}{2\pi}}$$

r	SA
0	∞
$\sqrt[3]{\dfrac{355}{2\pi}}$	277.5
∞	∞

So optimum dimensions are $r = \sqrt[3]{\dfrac{355}{2\pi}} \approx 3.837$,

$h = \dfrac{355}{\pi r^2} = 2\sqrt[3]{\dfrac{355}{2\pi}} \approx 7.674$ (in centimetres)

Actual dimensions: $r \approx 3.2$ cm, $h \approx 11$ cm