(1) Check if the expression can be re-written as a basic derivative.

Example: $\left(\dfrac{\square^2}{\square^5}\right)' = (\square^{-3})' \cdot \square' = -3\square^{-4} \cdot \square'$.

Warning: The power rule only works if the exponent is a constant. $(e^x)' \neq xe^{x-1}$. If it is a variable, then you must use the exponential rule: $(e^{\square})' = e^{\square} \cdot \square'$.

Warning: Don't forget to multiply by $\square'$. $(e^{x^2})' \neq e^{x^2}$ and $\left[(e^x)^2\right]' \neq 2e^x$. You can omit it only if $\square$ is $\boxed{x}$, in which case $\square' = 1$.

(2) If there is a constant in front of *every* term, pull it out. This is the Constant Times a Function Rule (p. 709).

Example: $(3x^2 + 3e^x)' = 3(x^2 + e^x)' = 3(2x + e^x)$.

Warning: The constant has to be in front of *every* term: $\left(-\dfrac{1}{x} + 1\right)' \neq -\left(\dfrac{1}{x} + 1\right)'$. If it's only in front of some terms, break up the derivative first (see Step 3).

(3) If the function is a combination, find the *last* combining operation.

Example: $y = x + e^x \rightsquigarrow$ last operation is addition.

Warning: $y = xe^x + \sqrt{x} \rightsquigarrow$ last operation is *not* multiplication, but addition.

(a) For addition or subtraction, use the Sum-or-Difference Rule (p. 711).

$$(u+v)' = u' + v'.$$

Example: $\left[x^2 + \ln(x)\right]' = (x^2)' + \left[\ln(x)\right]' = 2x + \dfrac{1}{x}$.

(b) For multiplication, use the Product Rule (p. 720):

$$(uv)' = u'v + uv' \quad \text{or} \quad (uv)' = u'v + v'u.$$

"Derivative of the first, times the second; plus derivative of the second, times the first." Order is *not* important.

Example: $\left[x^2 \ln(x)\right]' = (x^2)'\left[\ln(x)\right] + (x^2)\left[\ln(x)\right]' = 2x\ln(x) + x$.

Hint: Don't bother using the product rule for multiplication by a constant.

Warning: Don't just multiply the derivatives! $\left[x^2 \ln(x)\right]' \neq (x^2)'\left[\ln(x)\right]'$.

(c) For division, use the Quotient Rule (p. 722):

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \quad \text{or} \quad \left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}.$$

"Lo, d(hi), minus hi, d(lo), over lo squared." Order *is* important.

Example: $\left[\dfrac{x^2}{\ln(x)}\right]' = \dfrac{(x^2)'[\ln(x)] - (x^2)[\ln(x)]'}{[\ln(x)]^2} = \dfrac{2x\ln(x) - x}{[\ln(x)]^2}$.

Hint: Don't bother using the quotient rule for division by a constant.

Hint: You can use the product rule instead: $\left(\dfrac{e^x}{x^2}\right)' = \left(e^x x^{-2}\right)'$.

Warning: Don't just divide the derivatives! $\left[\dfrac{x^2}{\ln(x)}\right]' \neq \dfrac{(x^2)'}{[\ln(x)]'}$.

(d) For any other operation, use the Chain Rule (p. 731):

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}.$$

"Derivative of the outside, times derivative of the inside." This may be easier if you use box notation.

Example: If $y = \left[\ln(x)\right]^2$, then $y = u^2 = \square^2$, where $u = \square = \boxed{\ln(x)}$, so

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}u}(u^2) \cdot \frac{\mathrm{d}}{\mathrm{d}x}\left[\ln(x)\right] = \frac{2u}{x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\ln(x)}{x}.$$

Derivatives (starter kit)

y	y'
$\Box^n$	$n \cdot \Box^{n-1} \cdot \Box'$
$e^{\Box}$	$e^{\Box} \cdot \Box'$
$\ln(\Box)$	$\Box'/\Box$

Derivatives (exponential and logarithmic)

y	y'
$\ln\lvert\Box\rvert$	$\Box'/\Box$
$a^{\Box}$	$\ln(a) \cdot a^{\Box} \cdot \Box'$
$\log_a(\Box)$	$\Box'/[\ln(a) \cdot \Box]$
$\log_a\lvert\Box\rvert$	$1/[\ln(a) \cdot \Box]$

Derivative rules

(Constant-Multiple Rule) $$(c \cdot \Box)' = c \cdot \Box'$$

(Sum or Difference Rule) $$(\Box \pm \triangle)' = \Box' \pm \triangle'$$

(Product Rule) $$(\Box\triangle)' = \Box' \cdot \triangle + \Box \cdot \triangle'$$

(Quotient Rule) $$\left(\frac{\Box}{\triangle}\right)' = \frac{\Box' \cdot \triangle - \Box \cdot \triangle'}{\triangle^2}$$

(Chain Rule) $$\big[f(\Box)\big]' = f'(\Box) \cdot \Box'$$

(Chain Rule) $$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}.$$