Corrigendum to "Intersection homology with field coefficients: K-Witt spaces and K-Witt bordism"

Greg Friedman*

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The author's paper [2] concerns K-Witt spaces and, in particular, a computation of the bordism theory of such spaces. However, there is an error in the computation of the coefficient groups in dimensions $4k+2$ when $\mathrm{char}(K) = 2$. In this corrigendum, we state, as far as possible, the correct results. Details can be found in [1].

If we consider K-Witt spaces and K-Witt bordism using K-orientations, then for $\mathrm{char}(K) = 2$, this is unoriented bordism, which we denote $\mathcal{N}_*^{K-\mathrm{Witt}}$.

Theorem 1. *For a field K with char$(K) = 2$ and for*[1] $i \geq 0$,

$$\mathcal{N}_i^{K-Witt} \cong \begin{cases} \mathbb{Z}_2, & i \equiv 0 \mod 2, \\ 0, & i \equiv 1 \mod 2. \end{cases}$$

This result is also provided without detailed proof by Goresky in [3, page 498].

If we consider K-Witt spaces and K-Witt bordism using $\mathbb{Z}$-orientations, then we denote the bordism theory by $\Omega_*^{K-\mathrm{Witt}}$. In this case, there remains one ambiguity in the computation, but we can show the following:

Theorem 2. *For a field K with char$(K) = 2$ and $k \geq 0$,*

1. $\Omega_0^{K-Witt} \cong \mathbb{Z}$,
2. $\Omega_{4k}^{K-Witt} \cong \mathbb{Z}_2$,
3. $\Omega_{4k+1}^{K-Witt} = \Omega_{4k+3}^{K-Witt} = 0$,
4. *Either*
   - (a) $\Omega_{4k+2}^{K-Witt} = 0$ *for all k, or*
   - (b) *there exists some $N > 0$ such that $\Omega_{4k+2}^{K-Witt} = 0$ for all $k < N$ and $\Omega_{4k+2}^{K-Witt} \cong \mathbb{Z}_2$ for all $k \geq N$.*

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[1] Since these are geometric bordism groups, they vanish in negative degree.

See [1] for a discussion of the difficulty in determining which case of item (4) of Theorem 2 holds.

Independent of the existence or value of N in condition (4) of the theorem, the computations from [2, Section 4.5] of $\Omega_*^{K-\mathrm{Witt}}(\,\cdot\,)$ as a generalized homology theory on CW complexes continue to hold and to imply that for $\mathrm{char}(K) = 2$,

$$\Omega_n^{K-\mathrm{Witt}}(X) \cong \bigoplus_{r+s=n} H_r(X; \Omega_s^{K-\mathrm{Witt}}).$$

Similarly,

$$\mathcal{N}_n^{K-\mathrm{Witt}}(X) \cong \bigoplus_{r+s=n} H_r(X; \mathcal{N}_s^{K-\mathrm{Witt}}).$$

Other minor errata. In [2] it should not be part of the definition of a K-Witt space that the space be irreducible as a pseudomanifold. However, as every K-Witt space of dimension > 0 is bordant to an irreducible K-Witt space [4, page 1099], this error does not affect the bordism group computations of [2]. Not every 0-dimensional K-Witt space is bordant to an irreducible one, but the computation of $\Omega_0^{K-\mathrm{Witt}}$ reduces to the manifold theory and gives the result of [2] if one removes irreducibility from the definition.

The argument that $\Omega_{4k+2}^{K-\mathrm{Witt}} = 0$ given in [2] does not hold when $k = 0$. However, in this dimension it is not difficult to prove the result directly; details are provided in [1].

References

[1] Greg Friedman, *K-Witt bordism in characteristic 2*, preprint; see also http://faculty.tcu.edu/gfriedman.

[2] ______, *Intersection homology with field coefficients: K-Witt spaces and K-Witt bordism*, Comm. Pure Appl. Math. **62** (2009), 1265–1292.

[3] R. Mark Goresky, *Intersection homology operations*, Comment. Math. Helv. **59** (1984), no. 3, 485–505.

[4] P.H. Siegel, *Witt spaces: a geometric cycle theory for KO-homology at odd primes*, American J. Math. **110** (1934), 571–92.