Description of Lectures

Lecture 1. Introduction and Motivation. This lecture would focus on the basic structure of string theories (in rough terms suitable for mathematicians, not for an audience of particle theorists) and would discuss the idea of T-duality ("torus duality"), which posits an equivalence between a string theory on one spacetime manifold with a string theory on another spacetime, in which tori have been replaced by their duals ([12], [13], [1], [2], [20]). This duality is closely related to other dualities in string theory, such as mirror symmetry [33], which is proving to be a very useful concept in algebraic geometry, quite apparent from its importance in physics, and to the Fourier-Mukai transform [19], [5], [4].

Lecture 2. K-Theory and its Relevance to Physics. There is a substantial literature that suggests that "charges" in string theory should be represented not by ordinary integers but by elements of certain K-groups ([36], [37], [25], [29]). We will review some of the basics of topological K-theory, such as may be found in texts such as [21] and [15], and then discuss some of these arguments for the relevance of K-theory in physics.

Lecture 3. A Few Basics of C^*-Algebras and Crossed Products. This lecture will review some of the basics of C^*-algebra theory that will be needed in subsequent lectures. In particular, we will discuss the theory of C^*-crossed products, as developed, say, in the new book [35], including the Takai duality theorem [34].

Lecture 4. Continuous-Trace Algebras and Twisted K-Theory. We will review the definition of *continuous-trace* algebras, C^*-algebras locally Morita equivalent to commutative algebras, and the basic classification theory for them as developed by Dixmier and Douady [16] and the lecturer [32]. The K-theory of continuous-trace algebras is known as twisted K-theory and has been developed in [17], [32], [6], [3], and [22]. In the presence of background fluxes, the K-groups appearing in string theory must be replaced by twisted K-groups [9], [18].

Lecture 5. More on Crossed Products and Their K-Theory. This lecture will review some more aspects of the theory of C^*-crossed products (in the abelian case), concentrating in particular on Connes's "Thom isomorphism" theorem [14], the Pimsner-Voiculescu exact sequence [30], and properties of noncommutative tori. These results will be needed in subsequent lectures.

Lecture 6. The Topology of T-Duality and the Bunke-Schick Construction. This lecture will introduce the ideas of the papers [7], [8], and [11]. While T-duality for physicists is a *metric notion*, i.e., it depends on the metrics on the manifolds involved, we will concentrate in the next four lectures on just the *topological* aspect of T-duality, the part that is independent of the metric. This can be viewed as the "leading term" in the physicists' (metric) T-duality. The exciting discovery in [7] is that this leading term is highly non-trivial if one looks at non-trivial torus fibrations and/or the case of non-trivial background H-flux.

Lecture 7. T-Duality via Crossed Products. A curious discovery in [7] and [8] is that topological T-duality for principal circle bundles is formally analogous to a kind of duality for crossed products of actions of $\mathbf{R}$ on continuous-trace algebras, for which the action on the spectra is locally free but non-free. This duality had been discovered many years earlier in a totally different (purely C^*-algebraic) context in [31]. The analogy between the two dualities is not an accident; in fact, in joint work with Mathai Varghese, the lecturer showed that [31] can be exploited to reconstruct the results of [7] in a totally different way. We will explain the idea of the construction in this lecture.

Lecture 8. Higher-Dimensional T-Duality via Topological Methods. Bunke and Schick [11] showed that it is fruitful to consider the topological aspects of T-duality axiomatically and to attack them via methods of homotopy theory. In this lecture, we will discuss the results obtained this way in [28] and [10].

Lecture 9. Higher-Dimensional T-Duality via C^*-Algebraic Methods. In this lecture we will discuss the results on higher-dimensional T-duality obtained in [26], [28], and [27] via generalizations of the C^*-algebraic methods in [31]. A fascinating new phenomenon is that there are cases where the T-dual of a classical torus bundle is necessarily noncommutative, i.e., is basically a bundle of *noncommutative* tori over the base.

Lecture 10. Advanced Topics and Open Problems. In the last lecture, as time permits, we will discuss open problems and connections with different, but related, studies of mathematics coming from string duality, such as the study of the Fourier-Mukai transform in [19], [5], and [4]. Another example would concern recent suggestions (e.g., [23] and [24]) that, for some purposes, K-theory in string duality should be replaced by elliptic cohomology.

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