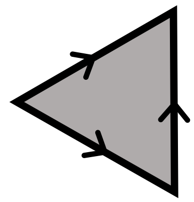
2019 Workshop in Geometric Topology
Problem Session
June 1, 2019

Craig Guilbault.

1) Does the Mazur compact contractible 4-manifold contain a pair of disjoint dunce hat spines?

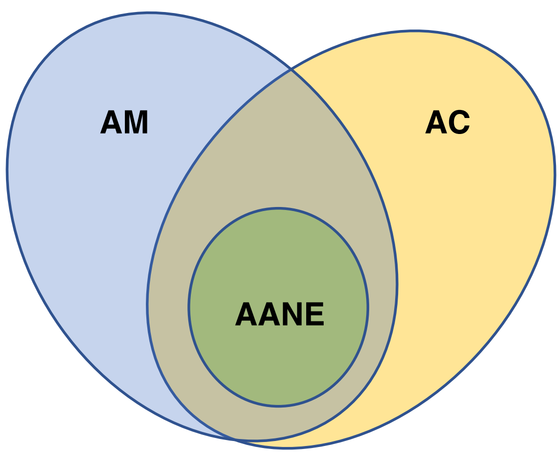
The *dunce hat* is the quotient space obtained from identifying the edges of the triangle as indicated.



2) Does any compact contractible 4-manifold ($\not\approx B^4$) contain a pair of disjoint spines?

Ric Ancel

In the class of metrizable spaces: let **AANE** denote the class of all approximate ANEs, **AC** the class of all approximate C-spaces, and **AM** the class of all approximately movable spaces. (Links to definitions are given below.) Consider the following Venn diagram:



Which of the regions **AM – AC**, **AC – AM**, **(AM ∩ AC) – AANE** is non-empty?
Which contains significant and/or interesting spaces?
Is there a sense in which **AANE** is a “negligible” subset of **AM** or **AC**?

Links to definitions: A space is a *C-space* if it has property C as defined in [W. Haver, “A covering property for metric spaces”, *Lecture Notes in Math.* **375** (1974) Springer, 108-113]. A space X is an *approximate C-space* if for every open cover $\mathcal{U}$ of X, there is a C-space Y and maps $u : X \to Y$ and $d : Y \to X$ such that $d \circ u : X \to X$ is $\mathcal{U}$-close to id_X. A space is an *ANE* if it is an *absolute neighborhood extensor for the class of metrizable spaces* as defined in [Hu, *Theory of Retracts,* Wayne State Univ. Press, 1965, p. 35]. A space X is an *approximate ANE* if if for every open cover $\mathcal{U}$ of X, there is an ANE Y and maps $u : X \to Y$ and $d : Y \to X$ such that $d \circ u : X \to X$ is $\mathcal{U}$-close to id_X. The definition of an *approximately movable space* is formulated in [Watanabe, "Approximative shape II", *Tsukuba J. Math* **11** (1987) 303-339].

Added after the Workshop: Kozlowski and Ancel have subsequently proved that AM = AANE.

Greg Friedman

Suppose L is a compact space and $L \times \mathbb{R}$ is an (n+1)-manifold. Is there an actual manifold M such that $L \times \mathbb{R}$ is homeomorphic to $M \times \mathbb{R}$? The answer is known to be "yes" for $n \geq 5$. What about the cases n = 3 and n = 4?

Nathan Sunukjian

Consider 2-knots $S^2 \cong K \hookrightarrow S^4$. Two 2-knots K_0 and $K_1 \subset S^4$ are *cobordant* if there a compact 3-manifold W^3 embedded in $S^4 \times [0,1]$ so that $\partial W^3 = (K_0 \times \{0\}) \cup (K_0 \times \{1\})$ and $\mathrm{int}(W^3) \subset S^4 \times (0,1)$. In the preceding definition, if W^3 is homeomorphic to $S^2 \times [0,1]$, then we say K_0 and K_1 are *concordant.* If, in addition, for each $t \in [0,1]$ which is a regular value of the embedding $W^3 \hookrightarrow S^4 \times [0,1]$ composed with the projection of $S^4 \times [0,1] \to [0,1]$, the level set $W^3 \cap (S^4 \times \{t\})$ is the union of a finitely many disjoint S^2's, then we say K_0 and K_1 are *0-cobordant* / *0-concordant*, respectively.

Facts: **1)** All 2-knots are concordant (and cobordant). (Kervaire)
2) Not all 2-knots are 0-concordant.

Question: Are all 2-knots 0-cobordant?

(If "yes", then exotic S^4's do not arise via Gluck twists.)

Qayum Khan

GENERAL QUESTION:

Let G be a compact Hausdorff topological group. Let M be a $\mathbb{Z}$-Cech cohomology manifold (in the sense of Borel's 1960 "Seminar on Transformation Groups"), equipped with a G-action by homeomorphisms. Let K be a compact subset of M.

Does K have only finitely many **G-orbit types** (i.e., G-conjugacy classes of stabilizers of points in K)?

AN IMPORTANT SPECIAL CASE:

Let T be a **protorus** (i.e., the inverse limit of toral groups), such as $T = (S^1)^J$ with the product topology for any set J. Let M be a $\mathbb{Z}$-cohomology manifold equipped with a T-action by homeomorphisms. Let K be a compact subset of M.

Does K have only finitely many T-orbit types (in this case, only finitely many subgroups of T are stabilizers of points in K)?

REMARK:

The general question was proven true when G is a compact Lie group by Floyd-Bredon in that seminar book. For any abelian compact Hausdorff group G, the connected component of the identity is a protorus, and the quotient group is abelian, totally disconnected, and compact (e.g., the p-adic integers $\mathbb{Z}_p = \lim_{\leftarrow} \mathbb{Z}/p^n\mathbb{Z}$, or more generally an abelian profinite group.)

Bob Daverman

Does there exist a closed manifold that regularly covers itself and the group of covering translations is non-abelian?

Atish Mitra

There is a longstanding open question of A.N. Dranishnikov: Is it true that the asymptotic dimension of a proper metric space equals the covering dimension of its Higson Corona? It is known that the covering dimension of the Higson corona is bounded above by the asymptotic dimension for every proper metric space, and that they are equal if the space has finite asymptotic dimension. Therefore, any proper metric space providing a counterexample to Dranishnikov's question should have infinite asymptotic dimension, whereas its Higson Corona should have finite covering dimension.

Asymptotic property C is a coarse analog of Haver's property C. It strictly weaker than finite asymptotic dimension: spaces with finite asymptotic dimension have asymptotic property C, but there are spaces of infinite asymptotic dimension with asymptotic property C.

Question: Consider a proper metric space of infinite asymptotic dimension with asymptotic property C. Is it true that the covering dimension of its Higson corona is infinite?

All the terms and concepts referred to above are defined in Bell and Dranishnikov's survey paper "Asymptotic dimension", *Topology Appl.* **155** (2008), 1265-1296.