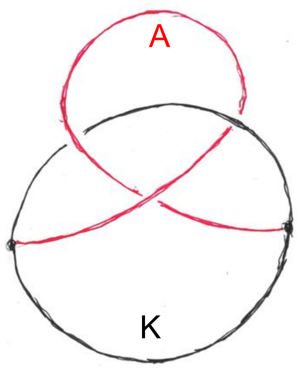
43rd Workshop in Geometric Topology – 2026, Utah Valley University

Problem Session – Saturday, June 20

Jack Calcut. [jcalcut@oberlin.edu] Let $\Theta \subset S^3$ be a theta curve. Assume Θ has an unknotted constituent knot K. Let $S^3 \to S^3$ be the double branched cover over K. The third arc A of Θ lifts to two arcs forming a knot J.



Folklore theorem of Thurston: Θ is prime $\Leftrightarrow$ J is prime (proved by Calcut and Metcalf-Burton in 2016).

(For the definition of *prime theta curve,* see Moriuchi, An enumeration of theta-curves with up to seven crossings, *J. Knot Theory Ramifications* **18** (2009), 167-197.)

QUESTION: What happens when we remove the hypothesis that Θ has an unknotted constituent knot?

Ashani Dasgupta. [ashani.dasgupta@cheenta.com] Lean 4's Mathlib is a machine-checked library of formal mathematics with about 620,000 declarations (definitions, theorems, lemmas, constructions) and 12.5 million logical dependencies. Form a graph whose vertices are the declarations, joining T_1 to T_2 when T_1 is used directly in the proof of T_2. Sampled measurements show two behaviors at two scales: the graph is globally thin (four-point Gromov $\delta \approx 1$) yet locally dense (a neighborhood-overlap proxy for Ollivier–Ricci curvature is positive, $\kappa \approx +0.02$). Call this a *tree of cliques*. Random graphs matching Mathlib's size and exact degree sequence do not reproduce the positive local curvature; it depends on which theorems depend on which, not on graph statistics alone.

The same structure appears independent of who wrote the mathematics. The sphere-packing formalization exists in two versions, one written by hand and one autoformalized by Math Inc's Gauss system, and both carry the same geometry: local curvature $\kappa \approx 0.0199$ (human) versus $\kappa \approx 0.0201$ (autoformalized), globally thin in both cases. The thin-and-fat signature is therefore not an artifact of human formalization style; it appears intrinsic to the mathematics.

QUESTION 1: Does this globally-thin, locally-dense structure persist as the library grows? That is, do δ and κ remain stable as Mathlib expands, and is the tree-of-cliques an approximate geometric feature of mathematics itself rather than of one library at one moment?

QUESTION 2: Can this geometry of mathematics be exploited predictively, to locate unproved conjectures and suggest possible proof strategies? For instance, do voids in a hyperbolic embedding of the graph mark missing theorems, and do geodesics between distant results trace usable proof paths?

For more information, see: A. Dasgupta, "Geometric Structure in the Lean 4 Mathlib Knowledge Graph"

Michael Bruner. [Michael@leebruner.com] A graph is said to be *n*-edge-bottlenecked if, for any two connected and disjoint subgraphs, there exists a collection of at most *n* edges such that any path from one subgraph to the other uses at least one of these edges. Bottlenecking defines a natural spectrum of graphs since every *n*-edge bottlenecked graph is also n'-edge-bottlenecked for any $n' \geq n$.

The classes of graphs with small edge bottlenecking have been the subject of interesting and fruitful study. These classes can admit a nice geometric characterization in terms of the interaction of cycles.

- The connected one-edge-bottlenecked graphs are trees; these are characterized by having no cycles.
- The two-edge-bottlenecked graphs are cactus graphs; these are characterized by having no intersection of distinct cycles larger than a single vertex.
- The three-edge-bottlenecked graphs are cut-cactus graphs; these are characterized by the intersection of any two cycles being empty or connected.

How nicely these classes and cycle-restriction characterizations extend each other leads to the question.

QUESTION: Can we extend the geometric/cycle restriction characterization of bottlenecked graphs further?

The spectrum of bottlenecked graphs naturally coarsens with the idea of fat-bottlenecking: an *M*-fat *n*-bottlenecked graph is M'-fat n'-bottlenecked for any $M' \geq M$ and $n' \geq n$. In 2024, a question was raised about extending some properties of the bottlenecking spectrum to a coarse setting; see that question for more details about fat/coarse bottlenecking.

For details on the bottlenecking spectrum and related concepts, see this preprint:

Bruner, M., Mitra, A. and Steiger H, Bottlenecking in Graphs and a Coarse Menger-type Theorem, arXiv:2406.07802v2 (2024).

Aditya DeSaha. [a.desaha@ufl.edu] If a group Γ acts freely and cocompactly on a space X, then (Brown) $H^*(\Gamma,\mathbb{Z}\Gamma) \approx H^*_c(X,\mathbb{Z})$.

QUESTION: What happens when you replace $\mathbb{Z}\Gamma$ by a general Γ-module M?
Is $H^*(\Gamma,M) \approx H^*_c(X,M^\Gamma)$?

Dan Margalit. [dan.margalit@vanderbilt.edu] Let S be a surface equipped with a hyperbolic metric. Say that three points are collinear if there is a geodesic segment containing all three. Here, a geodesic segment is an isometric immersion of an interval into S. Suppose $f : S \to S$ is a bijection that preserves collinearity of triples. Is it true that f is an isometry of S?

Ric Ancel. [ancel@uwm.edu] QUESTION 1: Is there a mismatch theorem for what happens to a triangulated n-manifold if you replace each n-simplex by a crumpled n-cube? In other words, is there a simple mismatch property that ensures that the resulting space is homeomorphic to the original manifold?

QUESTION 2: What happens to an n-dimensional theta curve (three n-balls with their boundaries identified) if you replace each n-ball by a crumpled n-cube? Are there non-trivial replacements that yield a theta curve?