2024 Workshop in Geometric Topology – Calvin College

Problem Session – Saturday, June 15

Michael Brunner. [Michael@leebruner.com] Question On Quasi-Bottlenecking
We define a graph to have n edge-bottlenecking if - for any two connected and disjoint subgraphs - there exists a collection of at most n edges such that any path from one subgraph to the other uses at least one of these edges. A graph is said to have M-fat n-bottlenecking at some scale M if - for any two connected M disjoint subgraphs - there exists a collection of n M-balls such that any path from one subgraph to the other intersects at least one of these balls. Finally, we say a graph is coarsely n-bottlenecked if there exists a scale M such that the graph is M-fat n-bottlenecked. Edge-bottlenecking defines a natural spectrum of graphs as all n-edge bottlenecked graphs are n′-edge bottlenecked for any n′ ≥ n. This spectrum naturally coarsens with the idea of fat-bottlenecking as a M-fat n-bottlenecked graph is M′-fat n′-bottlenecked for any M′ ≥ M and n′ ≥ n. This leads one to ask the natural question as to how the spectrum of bottlenecked graphs relates to the spectrum of coarsely bottlenecked graphs. Towards this end Bruner, Mitra and Steiger have proposed the following question.

QUESTION: (Conjecture 5.3.2 in [2]) If a graph is coarsely n-bottlenecked then is it quasi-isometricto a n-edge bottlenecked graph?

This question has a positive answer in the case of n = 1 and n = 2, these are a result of known classifications of quasi-trees and quasi-cactus. However even in the case of n = 2 the shortest proofs are several pages of technical work. Towards making these ideas more approachable we consider bottlenecked graphs in terms of excluded “ladders”, where we show that this idea of “ladders” coarsens in certain cases and explore the properties of “coarse skeletons” as a method for reducing the fatness of a bottlenecked graph. For details on bottlenecking and related concepts see our recent preprints [1, 2].

References

[1] Bruner, M., Mitra, A. and Steiger H., Graph Skeletons and Diminishing Minors., arXiv:2405.17785 (2024).

[2] Bruner, M., Mitra, A. and Steiger H, Bottlenecking in Graphs and a Coarse Menger-type Theorem, arXiv:2406.07802v2 (2024).

Yangyang Du. [yyadu@umich.edu] Two questions:
QUESTION 1: How does one classify Mazur manifolds (and Jester manifolds) with different framing by the fundamental group of their boundaries?

QUESTION 2: How does one classify Mazur manifolds (and Jester manifolds) with different framing by their hyperbolic volume?

Mathew Timm. [mtimm@bradley.edu] A question about self covers of 2-knots in S^4.

In “A Ribbon Knot Group Which has No Free Base,” Proceedings of the AMS, Volume 102, Number 4, April 1988, Katsuyuki Yosikawa introduces a family of smoothly embedded knotted 2-spheres in the 4- sphere. They are of interest to him interest because their knot complements have fundamental groups with deficiency 1 presentations which have no free base of finite rank. The fundamental groups for these 2-knot complements are also of interest because they are the generalized Baumslag-Solitar (GBS) groups

$$G = \langle x, y, t : x^p = y^q, tx^\alpha t^{-1} = y^\beta \rangle$$

where p, q, α, β are non-zero integers with “cross product” $p\beta - q\alpha = \pm 1$. Assume that all of p, q, α, β are positive.

These GBS groups have proper finite index subgroups which are isomorphic to themselves. Recall that a necessary condition for a manifold to have a *k*-fold self cover, for some $k \geq 2$, is for its fundamental group to have an index *k* subgroup isomorphic to itself. We therefore ask:

QUESTION: Do the knot complements associated to these GBS groups have a *k*-sheeted self cover for any $k \geq 2$?