35^{th} Workshop in Geometric Topology – 2018, Calvin College

Problem Session – Saturday, June 16

Greg Friedman. [greg.b.friedman@gmail.com] Given a map $\phi : X \to M_n(\mathbb{C})$ (the set of n $\times$ n matrices with complex entries) such that the eigenvalues of each $\phi(x)$ are distinct, consider the fiber bundle over X in which the fiber over each $x \in X$ is $\mathbb{C}^n$ is expressed as the direct sum of the 1-complex dimensional eigenspaces of $\phi(x)$. This bundle can be thought of as a *braided complex line bundle.*

QUESTION: Is there a name of these objects and are there known analogues of characteristic classes for them?

Craig Guilbault. [craigg@uwm.edu] QUESTION: Does the Mazur compact contractible 4-manifold contain a pair of disjoint of disjoint spines?

Bob Daverman. [rjdaverman@gmail.com] Let X be a subset of the n-sphere S^n.

QUESTION: When can a cell-like map $f : S^n \to S^n$ such that $f|X = id$ be approximated by a homeomorphism $h : S^n \to S^n$ such that $h|X = id$?

Remark: The answer is known to be "yes" if X is a knot in S^3.

Mike Mihalik. [michael.l.mihalik@Vanderbilt.Edu] QUESTION 1: Can the Pontryagin sphere be the boundary of a hyperbolic or a CAT(0) group?

Remark: After the Problem Session, several participants observed that the Davis construction applied to the cone over of compact orientable surface of positive genus can yield both hyperbolic and CAT(0) groups whose boundaries are Pontryagin spheres. Also, this construction is implicit in the methods of Dranishnikov's article "Boundaries of Coxeter groups and simplicial complexes with given links", *Journal of Pure and Applied Algebra* **137** (1999),139-151.

QUESTION 2: If G is a word hyperbolic group which is simply connected at ∞, must ∂G be simply connected?

Remark: The answer to Question 2 is "no" for CAT(0) groups. For example, if CK is the Croke-Kleiner group $< w, x, y, z \mid [w,x] = [x,y] = [y,z] = 1 >$, then $CK \times \mathbb{Z}$ is simply connected at ∞, but its boundaries are not simply connected.

Nathan Sunukjian. [nathan.sunukjian@calvin.edu] FACT: Every knot $S^1 \subset S^3$ can be unknotted by a sequence of crossing changes. Stated another way: every knot in S^3 can be unknotted by a sequence of ±1 surgeries on loops encircling a crossing.

QUESTION 1: Can every 2-knot $S^2 \subset S^4$ be unknotted by a sequence of log transforms on tori ($T^2 \times D^2 \circlearrowleft$ in S^4)?

QUESTION 2: Is this true for knotted tori T^2 in S^4?

Dubravko Ivansic. [divanic@murraystate.edu] QUESTION (and, by acclamation, worst math pun of the Workshop): How many sides does a *Muskegon* have?