

MATH 10043 CHAPTER 5 EXAMPLES AND DEFINITIONS

Section 5.2

RANDOM VARIABLE – a numerical variable whose value depends on the outcome of a random experiment. A random variable may be discrete or continuous.

Ex. A) Select a month at random. Are the following random variables discrete or continuous? What are some possible values for each? **(a)** n = the number of times it rained that month. **(b)** a = the amount of rainfall that month.

Ex. B) A randomly selected carton of 12 radios is inspected, and the number of defective radios is recorded. Identify the random variable used, and list its possible values.

PROBABILITY DISTRIBUTION – a distribution of the probabilities associated with each of the values of a random variable. This is a theoretical distribution, used to represent populations.

Ex. C) The number of people staying in a randomly selected room at a local hotel is a random variable; no more than 4 people are allowed in a room. The probability distribution is known by the hotel management, and appears below. If a room is selected at random, write in probability notation and find: **(a)** the probability that exactly two people are staying in the room. **(b)** the probability that the room is not empty.

x	0	1	2	3	4
$P(x)$	0.133	0.267	0.333	0.202	0.067

Notation to recall:

$\bar{x}$ -- sample mean s -- sample standard deviation
These are called **statistics**.

μ -- population mean [Greek letter lower-case 'mu']
 σ -- population standard deviation [Greek letter lower-case 'sigma']
These are called **parameters**.

Note: μ can also be called “**expected value**.”

Ex. D) Find the mean and standard deviation of x in the probability distribution constructed in example C.

Ex. E) Let x denote the grade level of a student chosen at random from the population data below. **(a)** Construct a probability distribution. **(b)** Find the mean and standard deviation of x . **(c)** Find the probability that the grade level of a student selected at random will be within one standard deviation of the mean.

GRADE	FREQUENCY
9	166
10	172
11	106
12	130
Σ	574

Practice Problems over Section 5.2

1. You are to survey randomly selected students at TCU with regard to the number of courses they are taking. **(a)** Identify the random variable of interest, **(b)** state whether it is discrete or continuous, and **(c)** list its possible values.

2. A barber shop has five chairs for waiting customers. The probability distribution for the number of customers waiting, y , is as given below. **Write in probability notation and find** the probability that: **(a)** exactly two customers are waiting. **(b)** at most four customers are waiting. **(c)** more than one customer is waiting. **(d)** between two and four customers are waiting, inclusive. **(e)** Find the mean and standard deviation of this probability distribution. **(f)** What is the probability that the number of waiting customers will be within one standard deviation of the mean?

y	0	1	2	3	4	5
$P(y)$	0.424	0.161	0.134	0.111	0.093	0.077

Section 5.3

BINOMIAL PROBABILITY EXPERIMENT

- Each trial has 2 possible outcomes (success and failure).
- There are n repeated, independent trials.
- $P(\text{success}) = p$ $P(\text{failure}) = q$ So, $p + q = 1$
- The **binomial** random variable x equals the **number of successes**.
- Probabilities must remain **constant** for each trial.

•**Finding a binomial probability using the calculator:**

2nd DISTR [this is the VARS key]
binompdf [option 0 or A] ENTER
enter values for n, p, x ENTER

Ex. F) It has been reported that about 28% of all résumés contain a major fabrication. Eighteen applicants for an actuarial position submitted résumés. **(a)** Find the probability that the number of résumés containing a major fabrication is exactly five. Write the answer as a complete sentence. Now find the probability that the number of résumés containing a major fabrication is **(b)** at most one. **(c)** more than one.

Ex. G) The United Nations Food and Agriculture Organization defines food security for a household as access by all members at all times to enough food for an active, healthy life. Many Texas residents do not qualify for food stamps, but are nevertheless food insecure. Within Tarrant County, 48% of food-insecure people do not qualify for food stamps. Suppose ten food-insecure people from Tarrant County are to be chosen at random. **(a)** Define a success. **(b)** list the possible values of the binomial variable x . **(c)** Find the probability that at most seven of them do not qualify for food stamps. **(d)** Find the probability that all of them qualify for food stamps.

Section 5.3 PRACTICE PROBLEMS

1. Studies show that 60% of U.S. families use physical aggression to “resolve” conflict. Suppose that 20 families are to be selected at random. Find the probability that the number of families that use physical aggression is exactly 5.
2. In January 2011, the Fort Worth Star-Telegram conducted a poll on legislative issues. Sixteen percent of those polled were in favor of increasing sales tax to close the deficit in the Texas budget. Suppose fifteen Fort Worth residents are sampled. What is the probability that at most three of them favor an increase in sales tax?
3. The TCU Office of Institutional Research reports that 71% of TCU undergraduates come from Texas. Twenty TCU undergraduates are selected at random. Find the probability that at most 17 of the undergraduates come from Texas.
4. Seventy-five percent of customers who sampled Tasty-Fry Frozen Onion Rings end up making a purchase. If 15 customers sample the onion rings, find the probability that all of them make a purchase. Write the answer as a complete sentence.

Section 5.4

MEAN AND STANDARD DEVIATION OF A BINOMIAL RANDOM VARIABLE

$$\mu = np$$

$$\sigma = \sqrt{npq}$$

Ex. H) Find the mean and standard deviation of the number of defective cellular telephones in a shipment of 500 if the manufacturer claims that 98.5% of the telephones are functioning properly.

Ex. I) A recent survey states that 74% of all U.S. drivers eat while driving. If ten U.S. drivers are to be chosen at random, find **(a)** the mean, and **(b)** the standard deviation of the number of U.S. drivers who eat while driving. **(c)** What is the probability that the number of U.S. drivers in this sample who eat while driving will be within one standard deviation of the mean?

Section 5.4 PRACTICE PROBLEMS

1. Nine percent of men have the type of colorblindness in which they cannot distinguish between the colors red and green. If sixteen men are to be chosen at random find **(a)** the mean, and **(b)** the standard deviation of the random variable x , where x = the number of men with this type of colorblindness.

2. It has been reported that 78% of Americans do some kind of reading for pleasure. **(a)** Out of a random sample of 20 Americans, what is the expected number of Americans who read for pleasure? [**Recall:** the mean is often called the expected value.]

(b) Find the standard deviation of the number of Americans who read for pleasure. Be sure to show how these numbers are calculated using the appropriate formulas.

3. According to a recent report, 42% of employees in the United States have elder care responsibilities. Suppose three U.S. employees are to be selected at random.

(a) Find the mean and standard deviation of the number of U.S. employees who have elder care responsibilities. **(b)** What is the probability that the number of U.S. employees who have elder care responsibilities will be within one standard deviation of the mean?