MATH 20043   MATH FOR ELEMENTARY TEACHERS
EXAM 1, FORM B   Chapters 1 & 2

Follow directions carefully. **SHOW ALL WORK**. Place your answers in the tables or spaces where given. Each problem is worth the number of points in brackets.

1. [7] (a) State whether the sequence is arithmetic, geometric, or neither. **(b)** State the common difference or common ratio if appropriate. **(c)** Find the missing terms of the sequence.

(c)

243,	81,	27 ,	9,	3,	1 ,	$\frac{1}{3}$

$\times\frac{1}{3}$ ($\div 3$) $\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$

(a) geometric

(b) $r = \frac{1}{3}$

2. [7] (a) State whether the sequence is arithmetic, geometric, or neither. **(b)** State the common difference or common ratio if appropriate. **(c)** Find the missing terms of the sequence.

(c)

6,	14 ,	22,	30,	38 ,	46 ,	54

$+8$ $+8$ $+8$ $+8$ $+8$ $+8$

(a) arithmetic

(b) $d = 8$

3. [6] Use the method of finite difference to find the next three terms of the sequence:

2, 6, 12, 20, 30, 42, _56_ , _72_ , _90_

$+4$ $+6$ $+8$ $+10$ $+12$ $+14$ $+16$ $+18$

$+2$ $+2$ $+2$ $+2$ $+2$ $+2$ $+2$

4. **[4]** List the first ten terms of the Fibonacci sequence.

$$1, \; 1, \; 2, \; 3, \; 5, \; 8, \; 13, \; 21, \; 34, \; 55$$

5. **[7]** In a parking lot there are motorcycles and cars. There are 132 tires and 39 engines. How many motorcycles and how many cars are in the parking lot? Write your answer as a <u>complete sentence</u>. [Note: there are no three-wheeled motorcycles.]

132 tires

39 engines = 39 vehicles

Let x = # of motorcycles

Then $39 - x$ = # of cars

$$2x + 4(39 - x) = 132$$
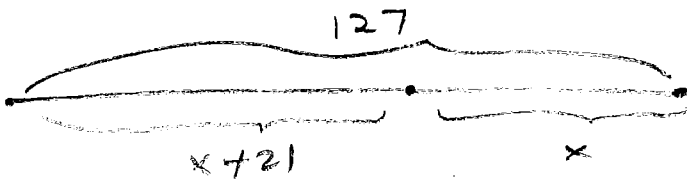
$$2x + 156 - 4x = 132$$

$$-2x = -24$$

$$12 = x$$

So $39 - 12 = 27$

There are 12 motorcycles and 27 cars in the parking lot.

6. **[3]** When two pieces of rope are placed end to end, their combined length is 127 feet. When the two pieces are placed side by side, one is 21 feet longer than the other. What are the lengths of the two pieces?

127

$x + 21$ x

$$x + 21 + x = 127$$

$$2x = 106$$

$$x = 53 \qquad x + 21 = 53 + 21 = 74$$

The ropes are 53 feet and 74 feet long.

2

7. [7] Classify each of the following as **true or false**. For each false answer, explain why it is false or correct the statement.

_F_ **(a)** The constant value added to the preceding term of an arithmetic sequence in called the common ~~ratio~~. _difference_

_F_ **(b)** The set of ~~whole~~ numbers is the set with elements {1, 2, 3, 4, . . .}. _natural_

_T_ **(c)** A false generalization occurs when one makes a universal statement based on too few facts.

_F_ **(d)** A function applies a rule to each element of the ~~range~~ _domain_ to give a single element in the ~~domain~~. _range_

_T_ **(e)** An important part of Polya's step _Understand the Problem_ is to be aware of one's assumptions.

_T_ **(f)** The origin of a coordinate system is the location where the two axes intersect.

_F_ **(g)** The ~~union~~ of two sets is the set of all elements common to both sets. _intersection_

8. [12] Let $U = \{M, A, T, H, I, S, F, U, N\}$ [U is the universal set for this problem]

$P = \{F, A, I, T, H\}$ $Q = \{S, H, I, N\}$ $R = \{N, U, T\}$

(a) List the elements in Q'.

$$Q' = \{M, A, T, F, U\}$$

(b) List the elements in $(P \cap Q) \cup R$

$$P \cap Q = \{I, H\}$$

$$(P \cap Q) \cup R = \{H, I, N, U, T\}$$

(c) List the elements in $P' \cap Q$.

$$P' = \{M, S, U, N\}$$

$$P' \cap Q = \{S, N\}$$

(d) List all the subsets of R.

$$\{N\} \quad \{U\} \quad \{T\}$$

$$\{N, U\} \quad \{N, T\} \quad \{U, T\}$$

$$\{N, U, T\} \quad \emptyset$$

9. **[6]** Let set **A** = {♣, ♦, ♥, ♠} and set **B** = {🍎, ✿, ❄}. Designate each of the following as **true or false**.

___T___ **(a)** {✿, 🍎} ⊆ **B**   ___F___ **(f)** **A** ⋃̸ **B** = ∅
                                                                        ∧

___F___ **(a)** {♦, ♣} ⊄ **A**   ___F___ **(b)** **B** ⊂ **B**   ⊆
                        ⊂

___T___ **(c)** ∅ ⊂ **A**   ___T___ **(d)** ❄ ∈ **B**
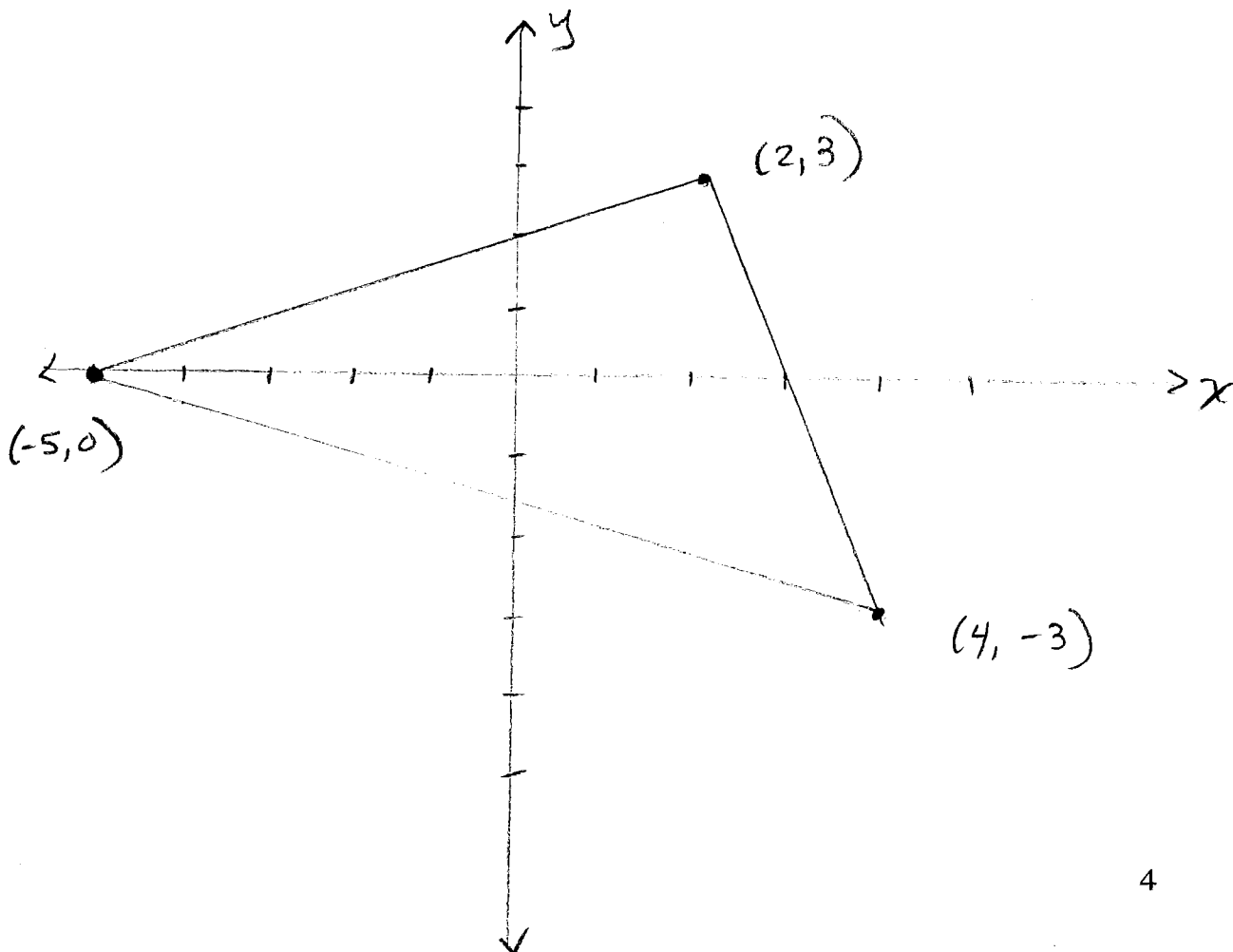
10. **[3]** If $f(x) = x^2 - 3x + 8$, find **(a)** $f(0)$, **(b)** $f(1)$, and **(c)** $f(-5)$.

(a) $f(0) = 0^2 - 3(0) + 8 = \boxed{8}$

(b) $f(1) = 1^2 - 3(1) + 8$
     $= 1 - 3 + 8 = \boxed{6}$

(c) $f(-5) = (-5)^2 - 3(-5) + 8$
     $= 25 + 15 + 8$
     $= \boxed{48}$

11. **[4]** Draw a triangle with vertices (4, -3), (2, 3), and (-5, 0). Label each vertex.


4

12. **[6]** Four children go to school along the same road. Erik jogged all the way. Nina rode her bicycle but stopped to talk to a friend. Sonya's father drove her to school. Tyler walked half the distance and then jogged the rest. The graphs below show distance as a function of time for each of these students. Determine which graph belongs to which student.
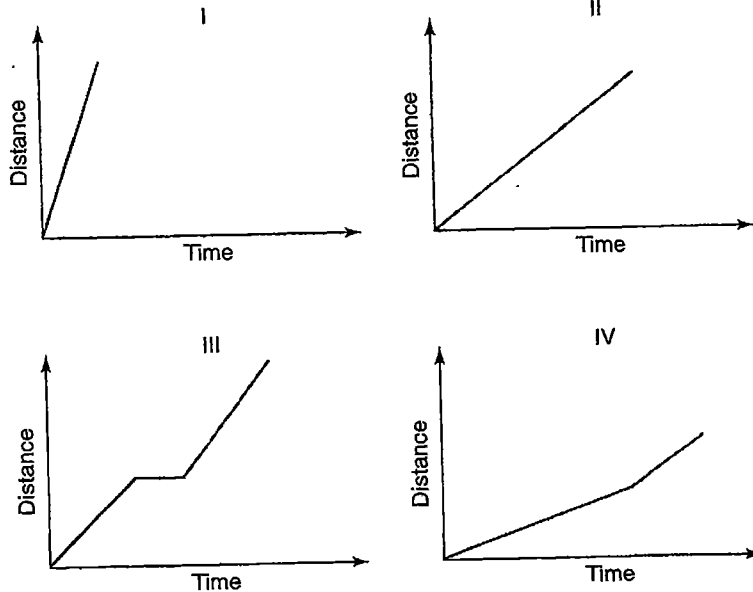
Erik   II

Nina   III

Sonya   I

Tyler   IV

13. **[8]** A group of students were surveyed: 37 are involved in a service organization, 29 are involved in a musical group, and 45 are involved in some kind of athletic activity. Sixteen of the students are involved in both service and music, 12 are involved in both music and athletics, and 14 are involved in both service and athletics. There are 6 students who are involved in all three. Use a Venn diagram to answer the following questions: **(a)** What is the total number of students surveyed? **(b)** How many students are involved in athletics only?
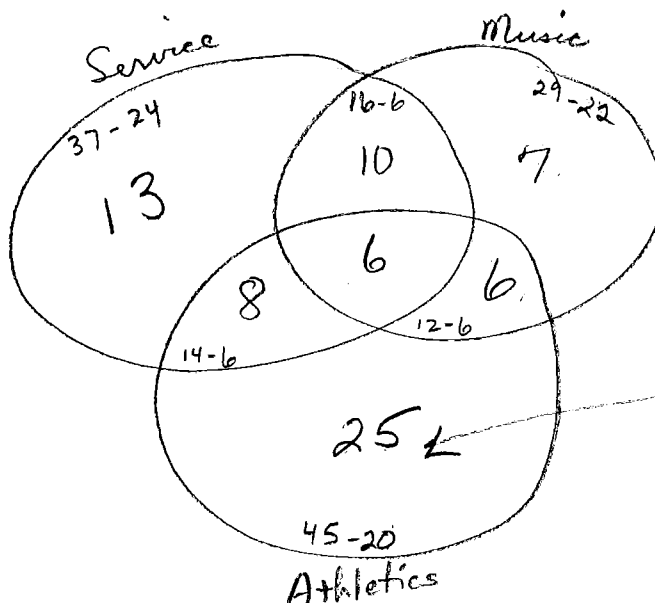
(a) $13 + 10 + 7 + 8$
$+ 6 + 6 + 25 = 75$
students surveyed

(b) 25 students are involved in athletics only.

14. [4] For the following, assume that the premises below are true. Draw a Venn diagram to determine whether the conclusion follows logically from the premises. State your decision and explain.

Premises: All giraffes like poetry. If you like pizza, then you like poetry. Alexander is a giraffe.

Conclusion: Alexander likes pizza.

Invalid conclusion.
Alexander may or may not
like pizza.

15. [5] For the following, assume that the premises below are true. Draw a Venn diagram to determine whether the conclusion follows logically from the premises. State your decision and explain.

Premises: All petunias are flowers.
Some flowers smell nice.

Conclusion: Some petunias smell nice.
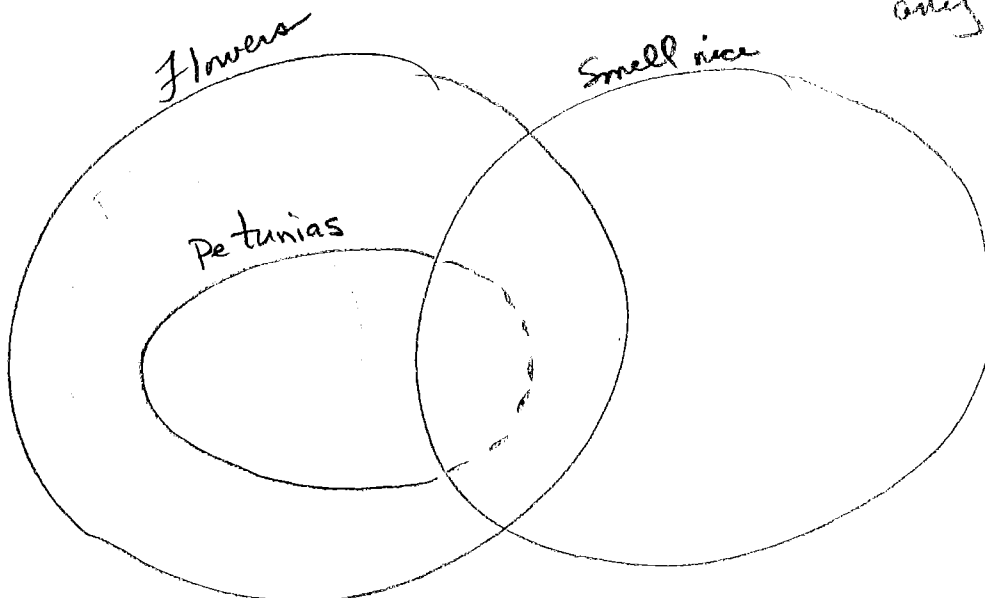
Invalid conclusion.
We do not know if
any petunias smell
nice.

16. **[11]** **Uneven matching**: Match the letter of the term with the correct definition below. NOTE: for consistency, all terms are singular, but may be plural in the definition. Terms are given in alphabetical order.

A – complement   **B** – coordinate   **C** – disjoint set   **D** – element

E – empty set   **F** – equal set   **G** – equivalent set   **H** – finite set

I – function   **J** – intersection   **K** – number sequence   **L** – premise

M – quantifier   **N** – open-ended   **O** – origin   **P** – range

Q – recursive   **R** – subset   **S** – union   **T** – universal set

_F_ **a)** Sets containing exactly the same elements.

_N_ **b)** A problem for which more than one answer is possible and/or more than one method can be used..

_A_ **c)** The _____ of set A is the set of all elements in the universal set U that are not in A.

_B_ **d)** A set of numbers called an ordered pair (x, y) indicating the horizontal and vertical location of a point in space.

_T_ **e)** The set of all elements being considered.

_L_ **f)** A set of facts used to draw a conclusion.

_Q_ **g)** Each successive term of the sequence is obtained from the previous term(s), at least after the first few terms.

_C_ **h)** Sets containing no elements in common.

_K_ **i)** A collection of numbers, called **terms**, arranged in order.

_G_ **j)** Sets containing the same number of elements.

_M_ **k)** Statements that include the words some, all, every, each, no, none.