MATH 20043 CHAPTER 4 EXAMPLES & DEFINITIONS

<u>Section 4.1 – *Factors & Multiples*</u>

Definition of Divisibility: If a and b are any whole numbers, then a is said to ***<u>divide</u>*** b if and only if there exists a **whole number** c such that b = ac. In other words, a ***divides*** b if and only if the quotient is a **whole number** — **no** remainder allowed!
NOTATION: **a | b "a divides b"**

For example: consider the fact $15 = 5 \cdot 3$
We can say:

5 divides 15	**also,** 5 is a divisor of 15
5 | 15	5 is a factor of 15
15 is divisible by 5	15 is a multiple of 5

DIVISIBILITY CRITERIA: A whole number x is divisible by:

2 if and only if the last digit of x is divisible by 2.

3 if and only if the sum of its digits is divisible by 3.

4 if and only if the last two digits of x are divisible by 4.

5 if and only if the last digit of x is 5 or 0.

6 if and only if x is divisible by both 2 and 3.

9 if and only if the sum of its digits is divisible by 9.

10 if and only if x ends in 0.

Ex. A) Use divisibility criteria to answer the following. John has 2,144 pieces of candy. Can he divide them evenly among:
(a) 2 people? (b) 3 people? (c) 4 people? (d) 5 people? (e) 6 people?
(f) 9 people? <u>Use divisibility criteria only!</u>

Definitions: A natural number greater than 1 is **prime** if and only if its only two distinct factors are 1 and itself. A natural number is **composite** if and only if it has a natural number divisor other than itself and 1.
NOTE: Since 1 has only one factor, it is considered neither prime nor composite.

Ex B) Determine whether 157 is prime.

Practice Problems Over Section 4.1

1. Leap years are divisible by 4. Years that end in two zeros must be divisible by 400 in order to be leap years. Which of the following are leap years?

(a) 1000 (b) 1492 (c) 1984 (d) 1800 (e) 1776 (f) 2006

2. Fill in each blank with "multiple" or "factor:"
   (a) 1 is a __________of every natural number.
   (b) 3 is a __________ of 15.
   (c) 25 is a __________ of 5.

Section 4.2 – *Prime Factorization*

Prime Factorization is the expression of a composite number as a product of all of its prime divisors. Two common methods for prime factorization are (1) the **prime-divisor method** (flowchart algorithm – see next page) and (2) the **factor-tree method**.

Ex. C) Work through the prime factorization flowchart algorithm to determine the prime factorization of 60. Then use the factor-tree method. What is an advantage and a disadvantage of each method?

Ex. D) Use a factor tree to find the prime factorization of 350.

Ex. E) Write the prime factorization of 1,386.

FUNDAMENTAL THEOREM OF ARITHMETIC:

Every composite natural number can be factored into a **unique** product of primes.

Practice Problems Over Section 4.2

1. Sketch a factor tree to find the prime factors of:
   (a) 90 (b) 320 (c) 495

Prime factorization flow chart:

start dividing by 2, if possible, for however many times possible, then by 3, then 5, etc.