MATH 20043 CHAPTER 3 EXAMPLES & DEFINITIONS

Section 3.1 *Numeration Systems*

Ex. A) Write as a Babylonian numeral:

(a) 14 **(b)** 74

Egyptian symbol	Corresponding whole number
Reed	One
Heel bone	Ten
Coiled rope	One hundred
Bent reed	One thousand
Pointed finger	Ten thousand
Burbot fish	One hundred thousand
Astonished man	One million

Egyptian hieroglyphs for whole numbers.

Ex. B) Write 110 as an Egyptian numeral.

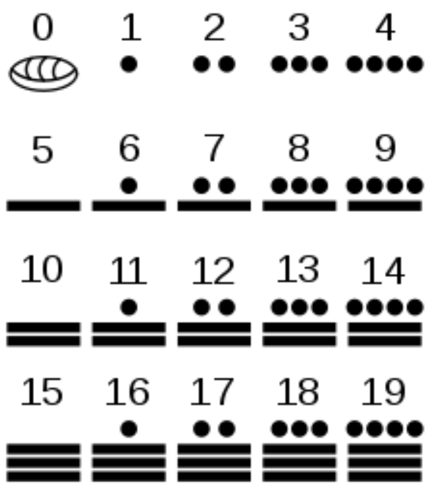
Chinese–Japanese Numerals and Their Values

Chinese–Japanese Numerals	Values	Chinese–Japanese Numerals	Values
一	1	七	7
二	2	八	8
三	3	九	9
四	4	十	10
五	5	百	100
六	6	千	1,000

2,347

二千	two 1,000s
三百	three 100s
四十	four 10s
七	7

Mayan numerals:



Ex. C) Write as a Mayan numeral:

(a) 38 **(b)** 67 **(c)** 127 **(d)** 300

Ex. D) Write as a Hindu-Arabic (base-ten) numeral:

(a) **(b)**

Greek Numerals and Their Values

Greek Numerals	Values
Ι (iota)	1
Γ (gamma)	5
Δ (delta)	10
Η (eta)	100
Χ (chi)	1,000
Μ (mu)	10,000

Roman numerals"

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

Some notes on using Roman numerals: There has never been a universally accepted set of rules for Roman numerals. Despite this, these rules have been frequently applied for the last few hundred years:

•The symbols "I", "X", "C", and "M" can be repeated three times in succession, but no more. "V", "L", and "D" cannot be repeated.

•"I" can be subtracted from "V" and "X" only. "X" can be subtracted from "L" and "C" only. "C" can be subtracted from "D" and "M" only. "V", "L", and "D" can never be subtracted. Only one small-value symbol may be subtracted from any large-value symbol.

•A horizontal line is used above a particular numeral to represent one thousand times that numeral, for example: $\overline{\text{V}}$ = 5,000

•A number written in Hindu-Arabic numerals can be broken into digits. For example, 2983 is made up of 2 thousands, 9 hundreds, 8 tens, and 3 ones. Thus 2,000 = MM, 900 = CM, 80 = LXXX, and 3 = III.
This gives us: 2983 = MMCMLXXXIII.

Ex. E) Write as a Roman numeral:
(a) 24 **(b)** 69 **(c)** 296 **(d)** 2013

Ex. F) Write as a Hindu-Arabic numeral:
(a) XIV **(b)** DCCCVIII **(c)** MCMXCIX

Ex. G) Write in expanded notation: thirteen thousand two hundred seven

Ex. H) Round 4,492,715 to the nearest:
(a) thousand **(b)** ten thousand
(c) hundred thousand **(d)** million

Practice Problems for Section 3.1

1. Convert from Roman numerals to Hindu-Arabic (base-ten) numerals:

(a) MCMLIX **(b)** XLIV **(c)** XLI
(d) CIII **(e)** MDCCLXXVI **(f)** CMXLVIII

2. Convert from Hindu-Arabic numerals to Roman numerals:

(a) 672 **(b)** 83 **(c)** 29 **(d)** 1998
(e) 1492 **(f)** 897 **(g)** 707

3. Convert from Hindu-Arabic numerals to Mayan numerals:

(a) 45 **(b)** 53 **(c)** 100 **(d)** 106 **(e)** 326

Section 3.2 *Addition and Subtraction*

NOTATION: If a + b = c, then a and b are called **addends,** and c is called the **sum.**

Ex. I) Demonstrate the addition of 5 and 4 using a number-line model.

ADDITION PROPERTIES

CLOSURE PROPERTY FOR ADDITION: For any two whole numbers a and b, there exists a **unique whole number** a + b.

COMMUTATIVE PROPERTY: If a and b are any whole numbers, then **a + b = b + a**.

ASSOCIATIVE PROPERTY: If a, b, and c are any whole numbers, then **(a + b) + c = a + (b + c)**.

IDENTITY PROPERTY: There is a unique whole number 0 (zero), called the **additive identity**, such that for any whole number a, **a + 0 = a** and **0 + a = a**.

Ex. J) John won the long jump by jumping 2 feet farther than his nearest competitor. If John jumped 7 feet, how far did his competitor jump?

ALGORITHM -- a pattern or procedure for performing a task. Often the method repeats some basic process several times. The term *algorithm* comes from the Persian al-Khuarizmi, who lived around 700 C.E. He is sometimes known as the Father of Algebra because of his book Al-Jabr wa'l muqabalah, from which comes our word 'algebra.'

Ex. K) **Demonstrate** the addition of 357 and 236 using **(a)** expanded notation, **(b)** a place value chart, **(c)** partial sums, and **(d)** the traditional algorithm.

Ex. L) **Demonstrate** the estimation of 7321 + 4847 + 3912 using: **(a)** rounding, and **(b)** front-end estimation.

SUBTRACTION: For any whole numbers a and b, **a – b = c** if and only if c + b = a.
NOTATION: If **a - b = c**, a is called a **minuend**, b is called a **subtrahend**, and c is called the **difference**.

Ex. M) **Demonstrate** the subtraction of 3 from 9 using a number-line model.

Ex. N) Thought question: When we read the word 'altogether' in a problem, are we referring to addition or subtraction? Consider the following problems:

1. A school collected 1,351 labels in a month. Suppose 1,649 more can labels are needed to get a computer. Altogether, how many labels will have to be collected?

2. The K – 3^{rd} grade classes turned in 1,351 soup labels this month. The school needs 3,000 labels to get a computer. How many labels will the 4^{th} & 5^{th} grade classes need to collect altogether for the school to get a computer?

Ex. O) Give a counterexample to show that subtraction is not associative under the whole numbers.

Ex. P) **Demonstrate** the subtraction of 752 and 396 using **(a)** expanded notation, and **(b)** the traditional algorithm.

INEQUALITIES

1. If a and b are any whole numbers, a is **less than** b if and only if there exists a natural number n such that a + n = b. **NOTATION**: **a < b**

2. If a and b are any whole numbers, a is **greater than** b if and only if there exists a natural number d such that a = b + d. **NOTATION**: **a > b**

More Notation: **a ≤ b** "a is **less than or equal to** b"
a ≥ b "a is **greater than or equal to** b"

Ex. Q) **(a)** Use the definition of less than to show that 3 < 8.
(b) Is 7 ≥ 4? Explain why or why not.

Practice Problems for Section 3.2 -- Addition

$\boxed{1.}$ Demonstrate the addition of 182 and 339 using **(a)** expanded notation, **(b)** a place value chart, and **(c)** the traditional algorithm.

$\boxed{2.}$ Demonstrate the estimation of 291 +438 + 665 using: **(a)** rounding, and **(b)** front-end estimation.

$\boxed{3.}$ A child made the same mistake in the two problems below. Use a complete sentence to explain the error made. 49 + 37 = 76 67 + 43 = 100.

Practice Problems for Section 3.2 -- Subtraction

$\boxed{4.}$ **Demonstrate** the subtraction of 814 and 396 using **(a)** expanded notation, and **(b)** the traditional algorithm.

$\boxed{5.}$ A child made the same mistake in the two problems below. Use a complete sentence to explain the error made. 92 – 39 = 67 408 – 322 = 126.

$\boxed{6.}$ A child made the same mistake in the two problems below. Use a complete sentence to explain the error made. 86 – 48 = 48 72 – 37 = 45.

$\boxed{7}$**.** Create an addition problem and a subtraction problem using Roman numerals. Demonstrate the solution for each.

Section 3.3 *Multiplication*

MULTIPLICATION: If a is a natural number and b is a whole number, then
$\mathbf{a \times b = b + b + \ldots + b}$ where b occurs a times.

NOTATION: If **a × b = c**, **a** and **b** are called **factors**, and **c** is called the **product**.

We can also write: **a · b = c** **ab = c** **a(b) = c** **(a)b = c**

MULTIPLICATION PROPERTIES

CLOSURE: For any whole numbers a and b, a × b is a **unique whole number**.

COMMUTATIVE PROPERTY: For any whole numbers a and b, **a × b = b × a**.

ASSOCIATIVE PROPERTY: For any whole numbers a, b, and c,

$$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times (\mathbf{b} \times \mathbf{c}).$$

IDENTITY PROPERTY: There is a unique whole number 1 (one) such that for any whole number a, $\mathbf{a} \times \mathbf{1} = \mathbf{a}$ and $\mathbf{1} \times \mathbf{a} = \mathbf{a}$.

ZERO PROPERTY: For any whole number a, $\mathbf{a} \times \mathbf{0} = \mathbf{0}$ and $\mathbf{0} \times \mathbf{a} = \mathbf{0}$.

Ex. R) **Demonstrate** the multiplication of 3 and 4 using: **(a)** repeated addition, **(b)** a number-line model, and **(d)** a square grid diagram (array model).

Ex. S) Martin reads his son 2 stories every morning and 4 stories every evening. By the end of the week, how many stories have they read?

ONE MORE MULTIPLICATION PROPERTY

DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION:

For any whole numbers a, b, and c, $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}.$

Also: $(\mathbf{b} + \mathbf{c}) \times \mathbf{a} = \mathbf{b} \times \mathbf{a} + \mathbf{c} \times \mathbf{a}.$

Ex. T) **Demonstrate** mental multiplication using the distributive property (expanded notation) by multiplying: 5 and 24.

Ex. U) **Demonstrate** multiplication using a square grid diagram (array model) by multiplying: **(a)** 5 and 24, **(b)** 35 and 26.

Ex. V) **Demonstrate** multiplication using the vertical format (partial products) by multiplying: **(a)** 5 and 24, **(b)** 35 and 26.

Ex. W) **Demonstrate** multiplication using the traditional method by multiplying: **(a)** 5 and 24, **(b)** 35 and 26.

Ex. X) **Demonstrate** mental multiplication using doubling and halving with 5×68.

Practice Problems Over Section 3.3

1. Demonstrate the multiplication of 2 and 6 using: **(a)** repeated addition, **(b)** a number-line model, and **(c)** a square grid diagram (array model).

$\boxed{2.}$ **Demonstrate** the multiplication of 28 and 47 **(a)** using vertical format (partial products), and **(b)** using the traditional method. **(c)** What is an advantage of each method?

$\boxed{3.}$ A student works out 25×12 as follows:

$$20 \times 10 = 200 \text{ and } 5 \times 2 = 10, \text{ so } 25 \times 12 = 210.$$

(a) What mistake was made? **(b)** Draw an array showing what the child left out.

$\boxed{4.}$ A child made the same mistake in the two problems below. Use a complete sentence to explain the error made. $36 \times 8 = 248$ $42 \times 6 = 242$.

Section 3.4 *Division and Order of Operations*

DIVISION: If a is a whole number and b is a natural number, then $a \div b$ equals c if and only if $c \cdot b = a$.

NOTATION: If $\mathbf{a \div b = c}$, then a is called the **dividend**, b is called the **divisor**, and c is called the **quotient.**

We can also write: $\mathbf{a/b = c}$ $\qquad \dfrac{\mathbf{a}}{\mathbf{b}} = \mathbf{c}$ $\qquad \mathbf{b\overline{)a}}^{\,\mathbf{c}}$

Ex. Y) Demonstrate the division fact $10 \div 2 = 5$ using: **(a)** repeated subtraction. **(b)** a number-line model. **(c)** a partition model.

Ex. Z) I want to divide my class of 42 students into 6 study groups. How many students will be in each study group?

Ex. AA) Which of the following are Division Properties for whole numbers?

Closure Commutative Associative Identity Distributive

Ex. AB) For a field trip, 82 children will be taken in vans that hold 10 children each. **(a)** How many vans will be needed? **(b)** Write a similar problem for which the answer is 8. **(c)** Write a similar problem for which the answer is 2.

DIVISION INVOLVING ZERO

- If $a \neq 0$, $0 \div a = 0$ because $0 \times a = 0$.

- If $a \neq 0$, $a \div 0$ is **undefined** because if $a \div 0 = d$, then $d \times 0 = a$, which is **false** for **any** value of d.

- $0 \div 0$ is **undefined** because if $0 \div 0 = c$, then $c \times 0 = 0$, which is **true** for **every** possible value of c. Thus, $0 \div 0$ has no unique quotient.

Ex. AC) Demonstrate division using scaffolding by dividing:
(a) 99 by 8, (b) 325 by 15.

Ex. AD) Demonstrate division using the intermediate format to find the quotient of 325 divided by 15.

Ex. AE) Demonstrate division using the traditional algorithm by dividing 325 by 15.

Ex. AF) Predict how most middle-school students would compute the following with no reminders of “order of operations.” $4 + 6 \times 5$

ORDER OF OPERATIONS

1. **( )** **2. exponents** **3. x ÷** (left to right) **4**. **+ -** (left to right)

Ex. AG) Evaluate: **(a)** $300 - 4\,(30 + 5) \div 2 \times 4$ **(b)** $60 - 20 \div 2^2 \times 5 + 8$

Ex. AH) (a) What is the purpose of grouping symbols? What are some grouping symbols? **(b)** Compute the following. For each of these, is the “order of operations” the best process? Explain. What can be done to alter the order? What allows us to do these alterations? **1.** $4\frac{2}{3} + 3\frac{1}{7} + 6\frac{1}{3}$ **2.** $25 \times 13 \times 4$

PRACTICE PROBLEMS OVER SECTION 3.4

$\boxed{1.}$ **(a)** For the division fact 6 ÷ 3 = 2, label each value. **Demonstrate** the division using: **(b)** repeated subtraction. **(c)** a number-line model. **(d)** a partition model.

$\boxed{2.}$ Seventy-two children were divided into seven teams of equal size. **(a)** How many children were on each team? **(b)** Were there any children who were not on a team? **(c)** Give some suggestions as to how many teams there should be so that each child will be on a team.

$\boxed{3.}$ **Demonstrate** 621 ÷ 22 using **(a)** scaffolding, **(b)** the intermediate method, and **(c)** the traditional algorithm. Make sure to show all steps!