

**Section 1.2 Patterns & Problem Solving**

**Number Sequence:** A collection of numbers arranged in order. Sequences are arranged from left to right, the numbers separated by commas.

**Recursive Pattern:** Each successive term of the sequence is obtained from the previous term(s), at least after the first few terms. To reveal a pattern, the sequence should contain at least 3 terms.

**Ex. A)** Find the next three terms of the sequence: 1, 5, 9, 13, . . .

**Arithmetic Sequence:** A sequence in which each term (after the first) is obtained by adding a constant value to the preceding term. That constant value is called the **common difference**.

**Ex. B)** Consider the sequence given. Is there a common difference? Find the next three terms. 16, 13, 10, . . .

**Geometric Sequence:** A sequence in which the ratio of each term to its predecessor is the same for all terms. **OR:** each term (after the first) is obtained by multiplying the preceding term by a constant value. That constant value is called the **common ratio**.

**Ex. C)** Find the common ratio and the next three terms of the following sequence. 2, 18, 162, . . .

**Ex. D)** Find an expression for the  $n^{\text{th}}$  term of a geometric sequence.

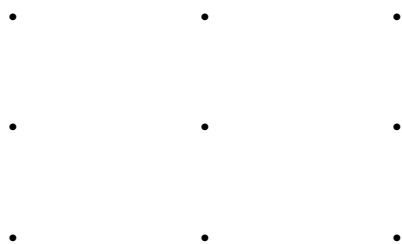
**Ex. E)** Find the next three terms of the sequence: 500, 100, 20, . . .

**Ex. F)** The sequence given is called the Fibonacci sequence. Find the pattern and the next three terms. 1, 1, 2, 3, 5, 8, 13, 21, 34, . . .

**Ex. G)** Find the pattern and the next term of the sequence: 1, 5, 14, 30, 55, . . .

## **Section 1.1 Introduction to Problem Solving**

**Ex. H)** Use a pencil to connect these dots with four straight line segments without lifting the pencil from the paper.



### **POLYA'S FOUR STEPS IN PROBLEM SOLVING -- OVERVIEW**

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|-----------------------------------|-------------------------------|
| 1. <i>Understand the problem.</i> | 3. <i>Carry out the plan.</i> |
| 2. <i>Devise a plan.</i>          | 4. <i>Look back.</i>          |

#### **1. UNDERSTAND THE PROBLEM**

- Reread the problem; state it in your own words.
- What are you trying to find? Identify the unknowns.
- Summarize the known information.
- Is unnecessary information given? Ignore irrelevant details. Watch out for your assumptions!

#### **2. DEVISE A PLAN: A few strategies**

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|--|--|
| <ol style="list-style-type: none"><li>Look for a pattern.</li><li>Eliminate impossibilities.</li><li>Look at related problems.</li><li>Try a simpler version of the problem.</li><li>Make a chart or table.</li><li>Draw a picture.</li><li>Choose an operation.</li><li>Guess, check, and revise.</li></ol> | <ol style="list-style-type: none"><li>Use a graph.</li><li>Write an equation (an algebraic model).</li><li>Work backward.</li><li>Try to find an intermediate goal rather than trying to solve the problem in a single step.</li><li>Take a break and try again.</li></ol> |
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#### **3. CARRY OUT THE PLAN**

- Implement your strategies; do any computations.
- Check as you go.
- \*\*\* Keep an accurate record of your work! In other words, show your work!

#### 4. LOOK BACK

- a. Check your results.
- b. Make sure that you have answered the original question. Does your answer **make sense**?
- c. In looking back, do you see another way you could have solved the problem?
- d. Could you use your strategy for other problems?

**Ex. I)** Twelve clothespins are placed on a line at 8-foot intervals. How far is the first from the last?

**Ex. J)** Jack goes to the well to get some water. He has a 3-liter pail and a 5-liter pail, and he wishes to return with exactly 1 liter of water. How can he do this?

**Ex. K)** Players on a football team are trying to gain weight. Tim, Jacob, Devon, and Carlos check their weights. Tim weighs twice as much as Jacob. Jacob weighs 50 pounds less than Devon weighs. Carlos weighs 10 pounds more than Devon weighs. If Carlos weighs 200 pounds, how much does Tim weigh?

**OPEN-ENDED PROBLEMS:** Problems for which students are asked to find more than one answer or use more than one method. Students may also be asked to design an extension to the problem.

**Ex. L)** Use the provided grid sheets to draw a shape with an area of:  
(a) 9 square units.                      (b) 12 square units.                      (c) 13 square units.

#### **PRACTICE PROBLEMS FOR SECTION 1.1** (solutions on last page)

1. The Browns have six daughters. Each daughter has two brothers. How many children are there in the Brown family?
2. [Classic Problem] Two ducks before a duck, two ducks behind a duck, and a duck in the middle are how many ducks?
3. Four boxes together weigh 60 pounds. Each box is twice as heavy as the next box, and the lightest box weighs 20 pounds less than the two middle ones together. What are the weights of the boxes? [Hint: try a table.]
4. How long will it take to cut a ten-foot log into five two-foot pieces, allowing two minutes per cut?

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### **PRACTICE PROBLEMS FOR SECTION 1.1, continued**

5. How can one use a three-minute timer and a five-minute timer to measure seven minutes? The timers are hourglass timers with no intermediate markings.
6. Can one determine which of ten dimes is counterfeit (lighter than the others) with only three weighings on a balance scale? How?
7. How can one obtain six gallons of cider from a tank if one has only a four-gallon container and a nine-gallon container? Neither container has any intermediate markings.

### **Section 1.3 Problem Solving with Algebra**

**Ex. M)** Kisha is 12 years old and her brother Alex is 2 years old. In how many years will Kisha be twice as old as Alex?

**Ex. N)** A loaf of bread and a gallon of milk together cost \$6.48. The milk costs 70¢ more than the bread. How much does each cost?

**Ex. O)** In a barnyard is an assortment of chickens and pigs. Counting heads, one finds 13; counting legs, one finds 46. How many pigs and how many chickens are there?

### **PRACTICE PROBLEMS FOR SECTION 1.3 (solutions on last page)**

1. Twenty-five coins total \$1. If one of the coins is a quarter, what are the other coins? [There is more than one correct answer, but you only need to find one.]
2. A short order cook earns \$8 an hour and her assistant earns \$5 an hour. During one long night they worked a total of 18 hours, and earned a total of \$129. How many hours did the short order cook work?
3. Lou drives a small car that gets 28 miles per gallon in the city and 34 miles per gallon on the highway. Her fuel tank holds 15 gallons. Regular gasoline sells for \$2.69 per gallon. What is the cost of a full tank of gasoline?
4. In a parking lot there are motorcycles and cars. There are 45 motors and 146 tires. How many motorcycles and how many cars are in the parking lot?

### **Solutions To Practice Problems, Section 1.1**

1. Since the girls share brothers, there are a total of 8 children in the Brown family.
2. Find the least number of ducks to satisfy the given conditions. The middle duck is used as part of the two ducks in “two ducks before a duck” and “two ducks behind a duck.” Thus there are 3 ducks. [Do not impose conditions that do not exist.]
3. The boxes weigh 4, 8, 16, and 32 pounds.
4. Multiply 4 cuts by 2 minutes per cut. It takes 8 minutes.
5. Start the three-minute timer and the five-minute timer simultaneously. At the end of three minutes, start the “official” seven-minute time period. Two minutes are left on the five-minute timer. After this, restart the five-minute timer. At the end, you will have timed seven minutes.
6. Separate the coins into two stacks of five each, and (1) weigh to determine the lighter stack. Separate the lighter stack into two stacks of two each, with one coin left out.  
(2) Weigh these two stacks. If they balance, the coin left out is the counterfeit. If they do not balance, take the lighter stack of two coins, and (3) weigh them against each other. The lighter of the two is the counterfeit coin.
7. Fill the four-gallon container twice from the nine-gallon container, leaving one gallon in the nine-gallon container. Empty the four-gallon container, and pour the gallon of cider into it. Now fill the nine-gallon container and fill the four-gallon container from it. [Note that since it had one gallon already, this takes three gallons out of the nine-gallon container.] Six gallons remain in the nine-gallon container.

### **Solutions To Practice Problems, Section 1.3**

1. One solution: 1 Q, 3 D, 6 N, 15 P. Another solution: 2 Q, 3 D, no N, 20 P.
2. The cook worked 13 hours.
3. We are looking for the cost of a full tank of gasoline. Multiply 15 gallons by \$ per gallon to get a cost of \$. [Note that we did not use the ‘miles per gallon’ information – it is irrelevant information.]
4. There are 28 cars and 17 motorcycles in the parking lot.