MATH 20043 **CHAPTER 2 EXAMPLES & DEFINITIONS**
FALL 2012

Section 2.1

SET – a collection of objects. A **well-defined set** is a collection of objects described in such a way that we can determine whether or not an object belongs to the set.

ELEMENTS – the objects in a set. **Notation**: $\in$.

NATURAL NUMBERS: The set composed of those numbers called the counting numbers. $\mathcal{N} = \{1, 2, 3, \ldots\}$

WHOLE NUMBERS: The set composed of the natural numbers and zero.
$\mathcal{W} = \{0, 1, 2, 3, \ldots\}$

Ex. A) Let E be the set of even natural numbers less than 10. List within braces the elements of set E.

EMPTY SET – a set containing no elements.

The **UNIVERSAL SET** is the set of all elements being considered.

The **COMPLEMENT** of set A is the set of all elements in the universal set U that are not in A. **Notation**: A' **Also**: $\overline{A}$

Ex. B) Let the universal set be the whole numbers from 0 to 10, and set E be the set of even natural numbers less than 10. List within braces the elements of set E'.

SUBSET: Set A is a **subset** of B if and only if each element of A is also an element of B. **Notation**: $A \subseteq B$

Ex. C) Use the definition of subset to answer the following: (a) Is a set a subset of itself? [HINT: Try a specific set such as S = {1, 2, 3}.] (b) Is the empty set a subset of set S?

Ex. D) Let M be the set of all mammals. Find a subset of M.

Ex. E) Let T = {a, b, c} Find all of the subsets of set T.

PROPER SUBSET: Set B is a **proper subset** of set A if and only if B is a subset of A, **and** B is **not equal to** A.
Notation: $B \subset A$

Ex. F) Let U (the universal set) consist of the letters of the alphabet, and A = {a, b, c}. For each of the statements below, choose the most precise of the following symbols to fill in the blank to make a true statement:

$$\in \quad \notin \quad \subset \quad \not\subset \quad \subseteq$$

(a) b ___________ A

(b) {c} __________ A

(c) {a, b, c} ____________ A

(d) d ___________ A

(e) {h} __________ A

(f) 0 ___________ $\varnothing$

EQUAL SETS: Two sets are **equal** if and only if they have exactly the same elements.

Ex. G) Construct a set D which is equal to set E (E = the set of even natural numbers less than 10).

A **ONE-TO-ONE CORRESPONDENCE** between sets A and B is a pairing of the elements in A and B such that for each element of A there is **exactly one** element of B, and for every element of B there is exactly one element of A.

Two sets A and B are **EQUIVALENT** if and only if there exists a one-to-one correspondence between the sets. **Notation**: **$A \sim B$**

Ex. H) Let P = {p, q, r}, Q = {q, r, p}, R = {5, 6, 7, 8}, and S = {5, 7, 8}. Which sets are equal? Which are equivalent?

An **INFINITE SET** contains an unlimited number of elements. A **FINITE SET** contains zero elements or a number of elements that can be stated as a specific natural number.

The **INTERSECTION** of sets A and B is the set of all elements common to **both** A and B. **Notation**: **$A \cap B$**

DISJOINT: If $A \cap B = \varnothing$, we say that A and B are **DISJOINT** sets.

Ex. I) Let M = {1, 2, 3, 4, 5} and N = {4, 5, 6, 7} Find $M \cap N$.

Ex. J) Draw a Venn diagram so that for sets A, B, and C, all the given conditions are satisfied: $A \cap B \neq \varnothing$, $B \cap C \neq \varnothing$, and $A \cap C = \varnothing$.

The **UNION** of sets A and B is the set of **all** elements in A **or** in B (or in both).
Notation: **$A \cup B$**

Ex. K) Let P = {a, b, c, d} and Q = {d, e, f}. Find $P \cup Q$.

Ex. L) Let A, B, and C be non-disjoint sets. Shade the region represented by:

(a) $A \cap (B \cap C)$. **(b)** $(A \cap B) \cap C'$.

Ex. M) In a class of 38 students, 20 students indicate that they exercise weekly, and 15 indicate that they get enough sleep per night. Five students said they exercise and get enough sleep. How many students neither exercise nor get enough sleep?

Ex. N) A discount store ran a special sale on calculators, spiral notebooks, and jeans. One salesperson reported that on one particular day, 50 customers took advantage of the sale. Thirty people purchased calculators; 25 purchased spiral notebooks; 35 purchased jeans; 14 purchased both calculators and spiral notebooks; 15 purchased both spiral notebooks and jeans; 22 purchased both calculators and jeans; 10 customers purchased all three. Was the salesperson's report accurate? Explain.

<u>Practice Problems Over Section 2.1</u>

1. (a) Let set R be the set of odd numbers less than 12. List within braces the elements of R.
(b) Let $T = \{x \mid x \text{ is a perfect square and } 0 < x \leq 100\}$. List within braces the elements of set T.
(c) Let the universal set be the natural numbers from 1 through 15, and let set A = {2, 5, 7, 8, 10, 24} List within braces the elements of set A′.
(d) Let the universal set be the letters of the alphabet, and set C be the set of consonants. List within braces the elements of set C′.

$\boxed{2.}$ For each of the statements below, choose one of the following symbols to fill in the blank to make a true statement: $\in$, $\notin$, $\subseteq$, $\subset$, $\not\subset$.
Be as precise as possible.

(a) 6 _______ {4, 5, 6, 7} **(b)** {5} ____________ {4, 5, 6, 7}

(c) {7, 9, 6} ________ {4, 5, 6, 7, 8, 9} **(d)** {4, 5, 6} ________ {4, 5, 6}

(e) {4, 5, 6} ________ {4, 5, 7} **(f)** 9 ________ {4, 5, 6, 7}

$\boxed{3.}$ Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, M = {1, 2, 3, 4, 5} and N = {4, 5, 6, 7}.
(a) Find $M \cup N$. **(b)** Find $N \cap M$.

>> continued <<

Practice Problems Over Section 2.1, continued

$\boxed{4.}$ A poll of 100 students was taken at a nonresidential college to find out how they got to campus. The results were as follows: 28 used car pools, 31 used buses, and 42 said that they drove to campus alone. In addition, 9 used both car pools and buses, 10 both used a car pool and sometimes drove alone, and 6 used buses and sometimes drove alone. Only 4 used all three methods. Draw a Venn diagram, labeling each of the relevant pieces. **(a)** How many students in the survey used none of the three transportation methods? **(b)** How many students used car pools exclusively? **(c)** How many students used buses exclusively?

Section 2.2

We can think of a **function** as a sort of machine that takes each element from a set called a **domain**, applies a rule to that element, and what comes out of the machine is a **single element** in a set called the **range**.

A **function** from set A to set B is a relation from A to B in which **each** element of A (the domain) is paired with **one and only one element** of B (the range).

Ex. O) Find the rule that transforms x into y:

x	1	2	3	4	5
y	2	5	10	17	26

Ex. P) Suppose that f(x) = 3x - 6. Find the following values: (a) f(2) (b) f(0)

Ex. Q) **(a)** Let f be a function that relates the length x of the side of a square to the area f(x) of the square. What is the equation of this function?
(b) Let g be a function that relates the length x of the side of a square to the perimeter g(x) of the square. What is the equation of this function?

Coordinates – a set of numbers called an ordered pair (x, y) indicating the horizontal and vertical location of a point in space.

Coordinate system -- (also called the **Cartesian coordinate system or the Rectangular coordinate system.**) The x axis is a horizontal number line that extends infinitely to the left and right; the y axis is a vertical number line that extends infinitely up and down. The two axes cross at right angles, and their point of intersection is the point 0 on each axis. This point is called the **origin**.

Ex. R) Locate the quadrants of the Cartesian coordinate system. Plot the following points:

(a) (-2, 3) **(b)** (5, -2)
(c) (1, 5) **(d)** (-3, -1)

Ex. S) (a) Graph the equation $f(x) = 3x + 2$. What does the graph of this equation look like? (b) Graph the equation $f(x) = \frac{1}{3}x - 2$. What does the graph of this equation look like? Plot 4 points to verify.

<u>Practice Problems Over Section 2.2</u>

1. Draw a triangle with vertices (5, -1), (-4, -1), and (-2, 6).

2. If $f(x) = x^2 - 3x + 4$, find **(a)** f(0), **(b)** f(1), and **(c)** f(-5) .

3. Sketch the graph of the equation $f(x) = -\frac{1}{2}x + 8$. Plot 4 points to verify.

<u>Section 2.3</u>

DEDUCTIVE REASONING: The process of reaching a necessary conclusion solely from a set of facts or hypotheses. These facts are called the **assumptions** or **premises**.

Ex. T) We went on a picnic today. Whenever I go on a picnic it rains.
What is the conclusion?

Ex. U) If this shape is a triangle, then it is not a square. **(a)** What is the premise? **(b)** What is the conclusion? **(c)** What conclusion can we draw if the premise is false?

Ex. V) Write an if-then statement with two assumptions.

Ex. W) Draw a Venn diagram, and draw a conclusion: All dolphins are whales. All whales are mammals.

Ex. X) **(a)** Draw a Venn diagram that represents the following statements.
All dolphins are swimmers.
All swimmers wear bathing suits.
No tigers wear bathing suits.

(b) Which of the following statements are confirmed by your diagram?
1. All dolphins wear bathing suits.
2. If you are not a tiger then you wear a bathing suit.
3. All dolphins are tigers.
4. If you are not a swimmer, then you are not a dolphin.

(c) What other valid deductions could we make?

FALSE GENERALIZATION: Generalizing from too few particulars that may not represent an entire group, or asserting a universal statement unsupported by evidence.

Ex. Y) Draw a Venn diagram to support or disprove the conclusion in the cartoon shown.

Ex. Z) Draw a Venn diagram to support or disprove the conclusion:
A pindel is a moofus. A lednip is a moofus. Therefore a pindel is a lednip.

Statements are **quantified** if they indicate “how many.” **QUANTIFIERS** include the words some, all, every, each, no, none.

Ex. AA) Use Venn diagrams to display the following quantified relationships:

(a) Some students are freshmen.
(b) Some flute players are oboe players.
(c) No birds are mammals.
(d) All mothers are females.

Practice Problems Over Section 2.3

1. Assume that the first two statements are true. Use a Venn diagram to answer the questions:

(a) All cats are elephants. Some elephants are red. Can we be certain that some cats are red?

(b) Carelessness always leads to accidents. I had an accident. Therefore I was careless. Does the third statement necessarily follow from the first two?

2. Use Venn diagrams to display the following quantified relationships:

(a) No cams are dills.
(b) Every boy is human and some boys are lazy.
(c) Some college students are athletes.

BONUS (optional) [1 point]: Use a Venn diagram to display the following relationship: All round murples are pink. Some square murples are pink.