Problem Set 2

Francis’ Quest of True Happiness

Overview: In this exciting problem set you will help Francis Edgeworth achieve true happiness (at least in a material sense and subject to his budget constraint). In particular, you will derive and graph his (iso-)utility curves. Francis has made things easier for you by getting himself stranded on a deserted island with only two goods; squid and bananas. Whew! 2 goods (i.e. 2 dimensions) sure makes things a lot easier. Thanks Francis!

Step 1: Getting to Know Francis’ Utility Function

Francis has two goods, squid (S) and bananas (B) available to him on this island. Francis’ utility function is as follows:

\[ U = S^{\alpha} B^{(1-\alpha)} \]

Pretty cool! We can calculate Francis’ utility if we know how much squid and bananas he consumes each day. But wait! To graph a (iso-)utility curve, we hold utility constant and figure out how much squid (S) is needed, given a certain amount of bananas (B), to maintain that utility level (U). We need squid (S) as a function of utility (U) and bananas (B). i.e. We need something of the form:

\[ S = f(U,B) \]

On page 3, fill in the specific equation for \( S = f(U,B) \). The rules of exponents below may help you. Show how you derived your specific equation for \( S = f(U,B) \) on page 3 as well.

Rules of Exponents:

\[ x^a x^b = x^{a+b} \quad \frac{x^a}{x^b} = x^{a-b} \]

\[ 1/x^a = x^{-a} \]

\[ (x^a)^b = x^{ab} \]

\[ (xy)^a = x^a y^a \]

\[ \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \]
Step 2: Plot Francis’ (Iso-) Utility Curves

It’s graphing time! On page 5, or on the spreadsheet available on-line, fill in the charts showing Squid (S) as a function of bananas (B) and Issac’s utility (U). Enter your #’s to at least 2 decimal places. You actually have four charts on the page. The values for α and U are given in the upper right of each chart. You have a column for utility (U) if you wish to plug the values of S and B back into the utility function to verify your numbers. Don’t worry about filling in the shaded and italicized cells. These data points won’t fit on the graph you will soon make … and they are already filled in for you.

Now graph and label each of the four curves on page 6. Alternatively, if you plugged your numbers for S into the spreadsheet (available on-line) it will make the graph for you. Simply print out the graph.

Step 3: Tradeoffs, Tradeoffs, Trade-offs

Life is about tradeoffs. Calculate the ratio at which Francis is willing to trade Squid for Bananas. Hmmm … This is the marginal Rate of Substitution (MRS) between Squid and Bananas (MRS_{S/B}). How the heck can we figure this out? Well, if an added banana gives Francis 5 times as much added utility as an added Squid (i.e. MU_B = 5 × MU_S), he is willing to trade 5 squid for 1 banana. His MRS_{S/B} = \frac{5S}{1B}.

In short, his MRS_{S/B} = \frac{\partial S}{\partial B} = \frac{MU_B}{MU_S} = \frac{\partial U}{\partial B} \frac{\partial B}{\partial S}

With this in mind, derive a specific formula for Francis’ Marginal Rate of Substitution. On page 3, fill in the specific equation Francis’ Marginal Rate of Substitution. Show how you derived this as well on page 3.

Using your formula for MRS_{F/C}, fill in the MRS columns on your charts on page 5.

Step 4: What Did I Just Calculate?

So far so good. Now let’s see if we can put it all together. Look at your MRS values and your (iso-)utility curves. What is the relationship between the MRS at a point and the slope of the (iso-)utility curves at that point? Describe this relationship on page 4.

On page 4, indicate what happens as α increases. Does an increase in α, ceteris paribus, mean that Francis favors squid more or that he favors bananas more? How is an increase in a reflected in his (iso-)utility curves?

Step 5: Wrapping It Up

Pick a new code name. Do not use the same code name you did in Problem Set 1. Put this code name on pages 3 - 6 of this problem set. Turn in pages 3 - 6.
Specific Equation for, and derivation of, S= f(U,B):

\[ \frac{\partial S}{\partial B} = \frac{\partial U}{\partial S} \]

Specific Equation for, and derivation of, \( MRS_{S/B} \), i.e.

\[ \left. \frac{\partial S}{\partial B} \right|_U = \frac{MU_B}{MU_S} = \frac{\partial U}{\partial B} \]
Relationship between MRS at a point and the slope of the (iso-)utility curve at that point:

Does an increase in $\alpha$, ceteris paribus, mean that Francis favors squid more or that he favors bananas more? What does “ceteris paribus” mean in this instance? How is this reflected in the (iso-)utility curves?
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