Characteristics of the Cobb-Douglas Production Function

Overview: In this problem set you will demonstrate the many desirable characteristics of the Cobb-Douglas production function. The Cobb-Douglas production function, for a model of the economy with only Capital (K) and Labor (L) as inputs is:

\[ Y = K^{\alpha} \times L^{(1-\alpha)} \quad \text{Where } 0 < \alpha < 1 \]

For this problem set, assume that \( \alpha = 0.4 \)

Instructions

1. Demonstrate that the Cobb-Douglas production function is a constant returns to scale (CRS) production function. Do this by:
   a. Calculating \( Y \) when \( K = 20 \) and \( L = 40 \)
   b. Calculating \( Y \) when \( K = 40 \) and \( L = 80 \)
   c. Calculating \( Y \) when \( K = 100 \) and \( L = 200 \)
   d. Explain what constant returns to scale is. Explain how your numbers demonstrate, or fail to demonstrate, that the Cobb-Douglas function is CRS.

2. Demonstrate that the Cobb-Douglas production function exhibits diminishing marginal productivity of labor. Do this by:
   a. Hold \( K \) constant at 10. Vary \( L \), in increments of 1 from 1 to 8. Calculate \( Y \) for each \( L \). Then calculate the added output per added unit of Labor, i.e. MPL. Fill in the chart below.

\[
\begin{array}{cccc}
K & L & Y & MPL = \frac{\Delta Y}{\Delta L} \\
10 & 0 & 0 & NA \\
10 & 1 & & \\
10 & 2 & & \\
10 & 3 & & \\
10 & 4 & & \\
10 & 5 & & \\
10 & 6 & & \\
10 & 7 & & \\
10 & 8 & & \\
\end{array}
\]
b. Explain what diminishing marginal productivity of labor is. Explain how your numbers demonstrate, or fail to demonstrate, that the Cobb-Douglas function exhibits diminishing marginal productivity of labor.

3. Demonstrate that the Cobb-Douglas production function exhibits diminishing marginal productivity of Capital. Do this by:
   a. Hold L constant at 10. Vary K, in increments of 1 from 1 to 8. Calculate Y for each L. Then calculate the added output per added unit of Labor, i.e. MPK. Fill in the chart below.

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b. Explain what diminishing marginal productivity of capital is. Explain how your numbers demonstrate, or fail to demonstrate, that the Cobb-Douglas function exhibits diminishing marginal productivity of capital.

4. Demonstrate that the Cobb-Douglas production function abides by Euler’s theorem. Do this by:
   a. Explain what Euler’s theorem is and how it applies to production function. In particular, list the conditions (CRS, etc.) required for Euler’s theorem to hold. The state the outcome (Y = something) according to Euler’s theorem.
   b. Let K = 100 and L = 100. Calculate Y.
   c. Calculate the Marginal product of labor (MPL). To do this calculate Y when K = 100 and L = 99. The added output when L increases to 100, calculated in part b, is the MPL.
   d. Calculate the Marginal product of capital (MPK). To do this calculate Y when K = 99 and L = 100. The added output when K increases to 100, calculated in part b, is the MPK.
   e. Calculate K×MPK + L×MPL. Use the MPL and MPK you calculated in c and d respectively. Use 100 for K, and 100 for L. Does this equal Y when K = 100 and L = 100?1

1 Note: your results may be off by a tiny bit because we calculated MPL and MPK based on discrete changes instead of using calculus.