1. For $S = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : |x| + |y| \leq 22\}$, find $|S|$.

2. A pizza parlor has 8 toppings (sausage, pepperoni, hamburger, onions, black olives, green peppers, garlic, and mushrooms).
   (a) How many ways to order a pizza with 4 different toppings?
   (b) How many ways if one topping can be doubled?
      (for example, double sausage, onions and black olives)

3. Consider 5 card hands from a standard deck of 52 cards.
   (a) How many 5 card hands are possible?
   (b) How many full houses? A full house consists of two of one kind and three of another, such as (2♣, 2♣, 2♣, 6♥, 6♥).
   (c) How many flushes? A flush has all cards of the same suit.
   (d) How many three of a kind?

4. A box contains 20 marbles, 4 are blue, 7 red, and 9 yellow.
   (a) How many ways to choose 6 marbles from the box?
   (b) How many ways to choose 6 yellow marbles?
   (c) How many ways to choose 6 with 4 red and 2 blue?

5. Consider integers $1 \leq n \leq 25,000$.
   (a) How many are multiples of 3, 5 or 7?
   (b) How many are multiples of 3 or 5 but not 7?

6. How many lists of integers $(a, b, c)$ are there with $1 \leq a, b, c \leq 10$ and $abc$ even? (repetition is allowed)

7. Find the number of ways to color the squares of a $4 \times 4$ checkerboard with red and yellow so that no row is entirely one color.

8. A professor asks her students about computer science and math courses taken. Fifty students took the survey and she found:
   - 30 took precalculus
   - 18 took calculus
   - 26 took Java
   - 16 took both precalculus and Java
   - 8 took both calculus and Java
   - 9 took both precalculus and calculus
   - 47 took at least one of the three courses.
   (a) How many students did not take any of the three courses?
   (b) How many students took all three courses?
   (c) How many took precalculus and calculus, but not Java?
   (d) How many took precalculus, but no calculus or Java?
Hints

1. Draw a picture like we did in class on February 27.
2. For part (b), add to your answer from (a) the number of ways with a doubled topping.
3. Parts (b) and (c). What choices do you need to determine a full house or a flush? (d) Make sure you don’t accidentally include full houses or four of a kind in your count.
4. Part (a) is much like an example from class. For part (b) you will find it helpful to draw a Venn diagram.
5. Observe that $abc$ is even if and only if at least one of $a, b, c$ is even. Now use inclusion-exclusion formula.
6. Use inclusion-exclusion to determine how many colorings have at least one row of the same color.
7. (a) Easy if you read question fully. (b) Use inclusion-exclusion. (c) and (d) It is helpful to draw the Venn diagram.