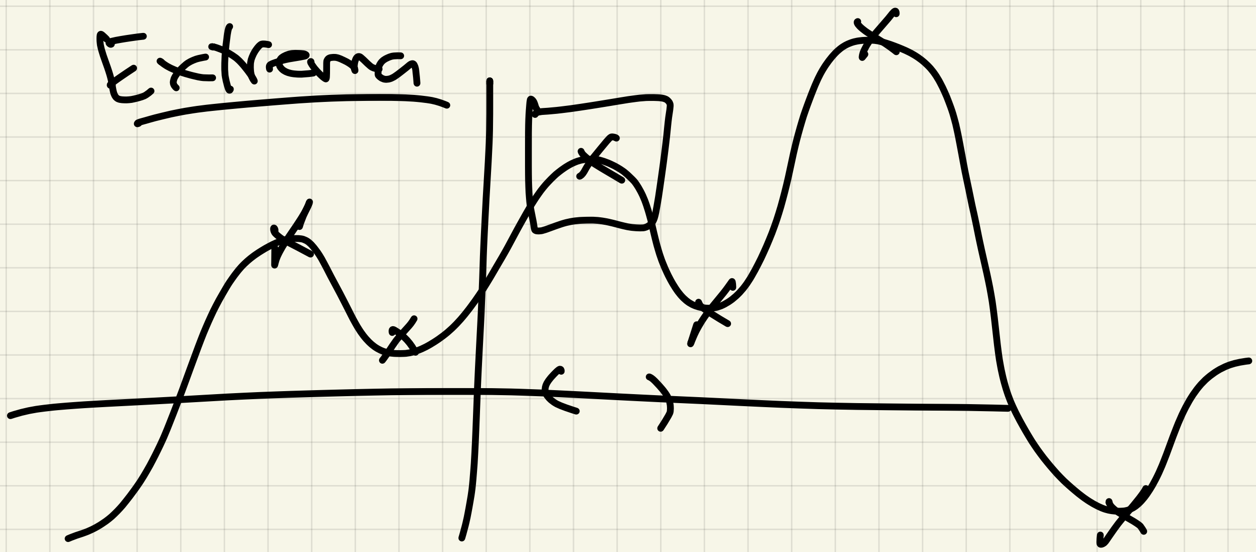
3/8 | Calc I   Exam 2 → 3/21

Hw 14b w/ in exam

4.1 Extrema



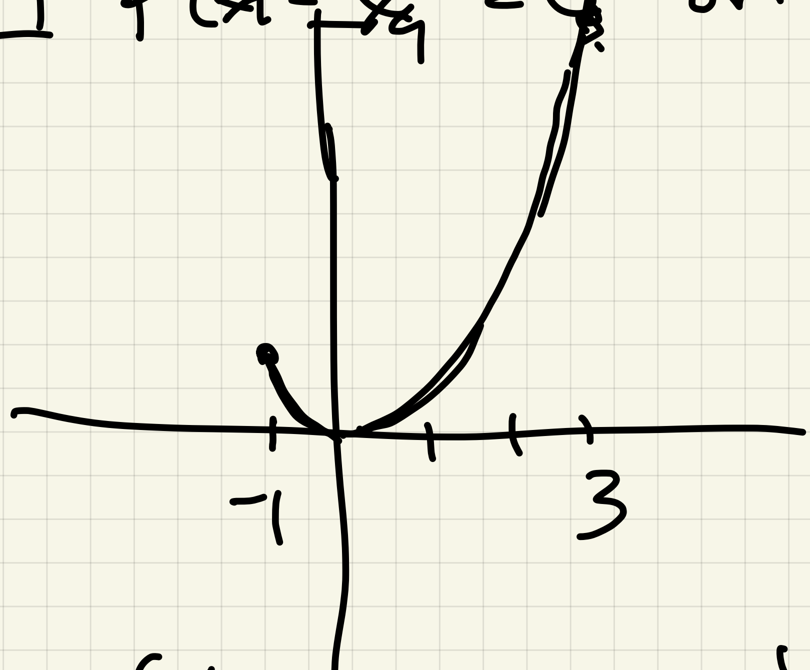
Defn If $y = f(x)$ is a function on an interval I containing c, then

① $f(c)$ is the absolute maximum of f on I if $f(c) \geq f(x)$ for all x in I

② $f(c)$ is the absolute minimum of f on I if

$f(c) \le f(x)$ all x in I

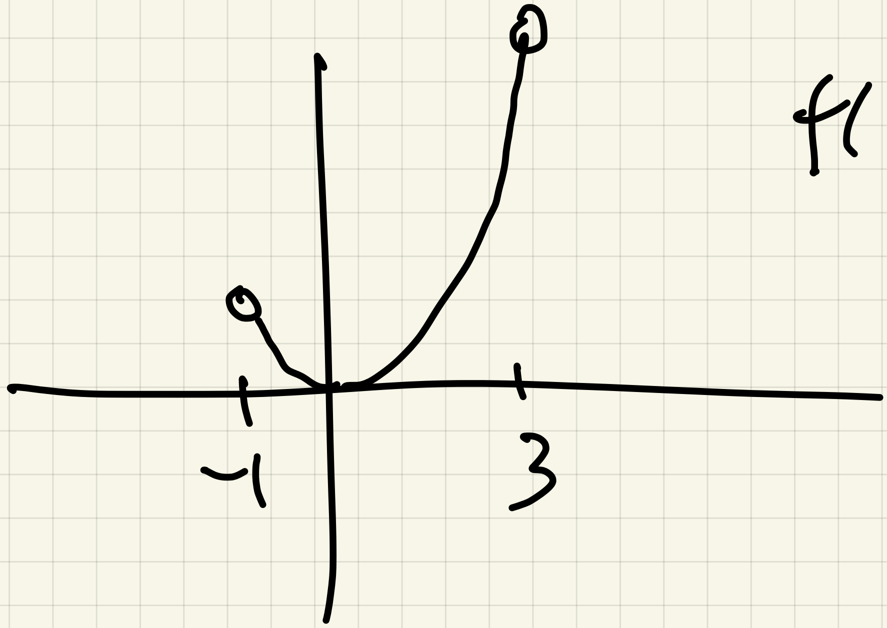
<u>Ex</u> $f(x) = x^2 = y$ on $I = [-1, 3]$



$f(0) = 0$ is <u>abs min</u>

$f(3) = 9$ is <u>abs max</u>

<u>Ex</u> $f(x) = x^2$ on $(-1, 3)$

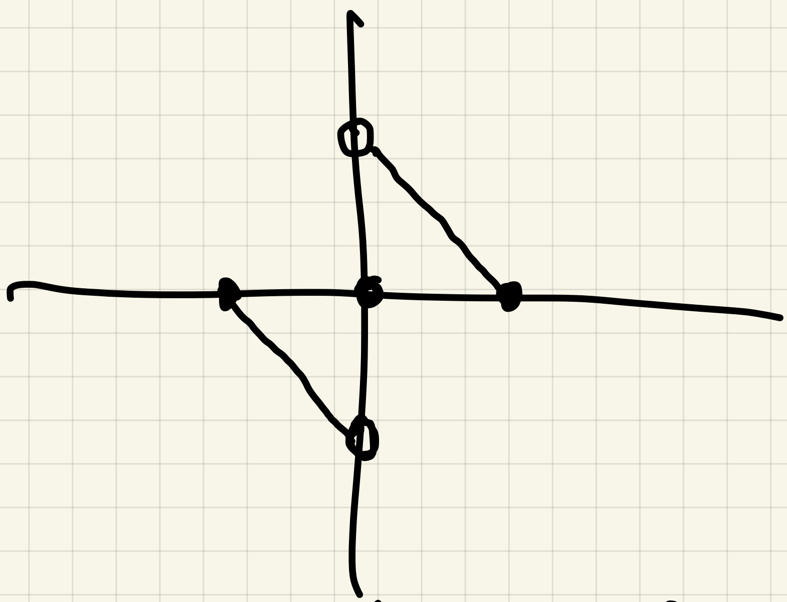


$f(0) = 0$ <u>abs min</u>

No <u>abs max</u> !!

Ex3

$$f(x) = \begin{cases} -x-1 & -1 \le x < 0 \\ 0 & 0 \\ 1-x & 0 < x \le 1 \end{cases}$$



No abs min!

No abs max

Thm1 If $y = f(x)$ is continuous on a closed interval $I = [a,b]$, then $f(x)$ has both abs max and abs min.

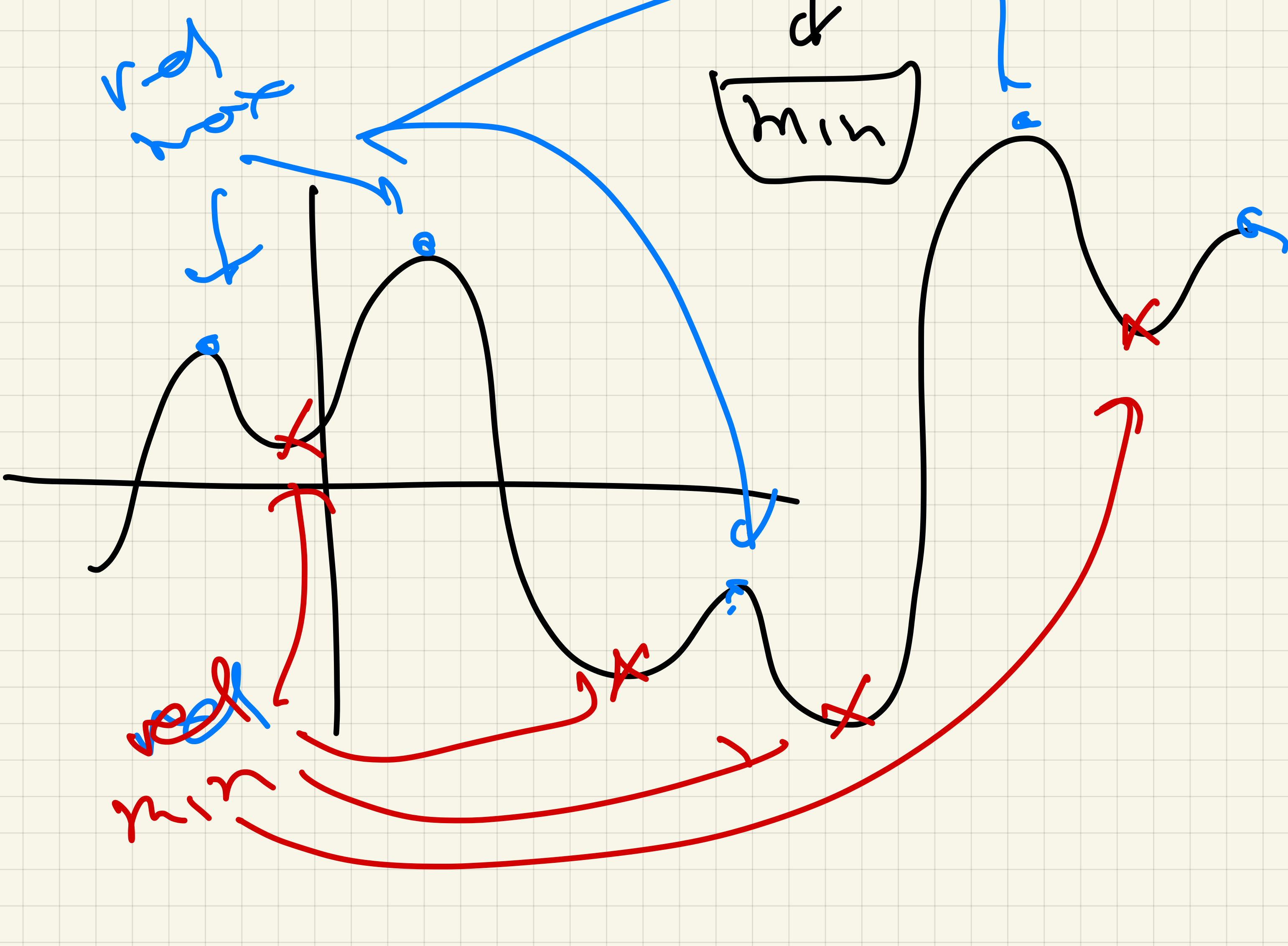
Defn:

① $f(c)$ is a relative maximum (local max) if there is

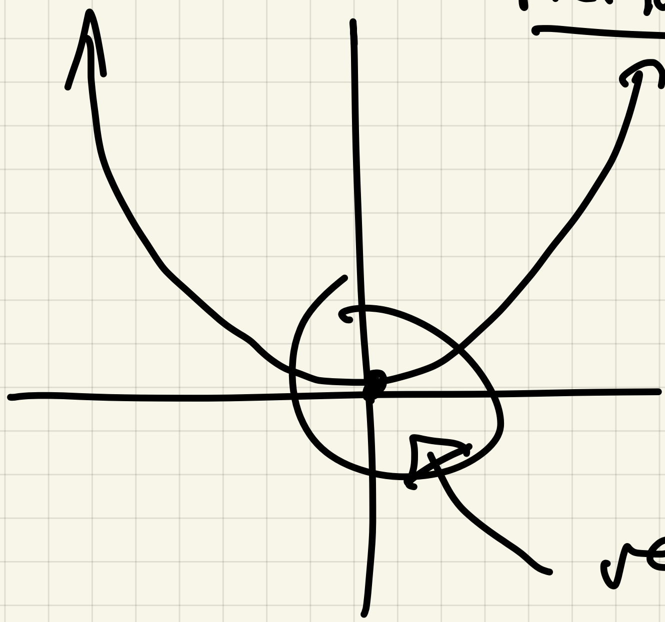
an open interval I
containing c so that
$f(c)$ is abs $\boxed{\text{max}}$ on I

② Similarly $f(c)$ is a
<u>relative minimum</u> (<u>local min</u>)

if . . ___



Ex 4 $f(x) = x^2$ has rel min at $x = 0$

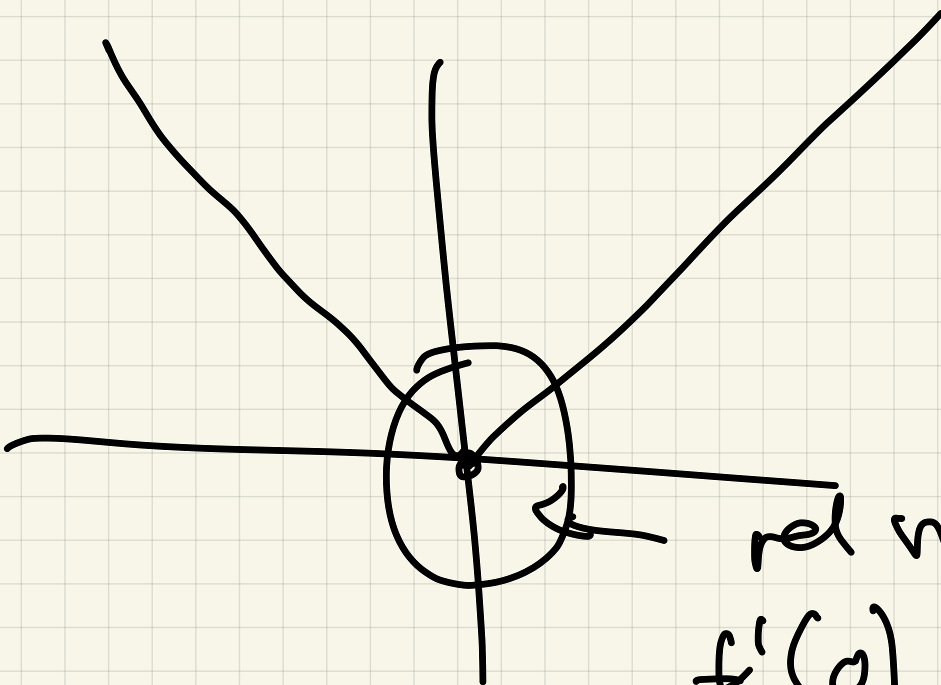


no rel max

rel min

$f'(0) = 0$

Ex 5 $f(x) = |x|$



rel min

$f'(0) = DNE$

Defn c is a <u>critical point</u> for $y = f(x)$ if

$f'(c) = 0$ or $f'(c)$ DNE

Thm If $f(x)$ has a <u>rel max/ rel min</u> at $x = c$, then c is a critical point for $f(x)$.

Ex6 Find all critical points for $\boxed{f(x) = x^4 - x^2}$, on $\mathbb{R}$ and all rel max/mins, and abs max/mins.

$f'(x) = 4x^3 - 2x$ (always exists)

$$4x^3 - 2x = 0$$

$$2x(2x^2-1) = 0$$

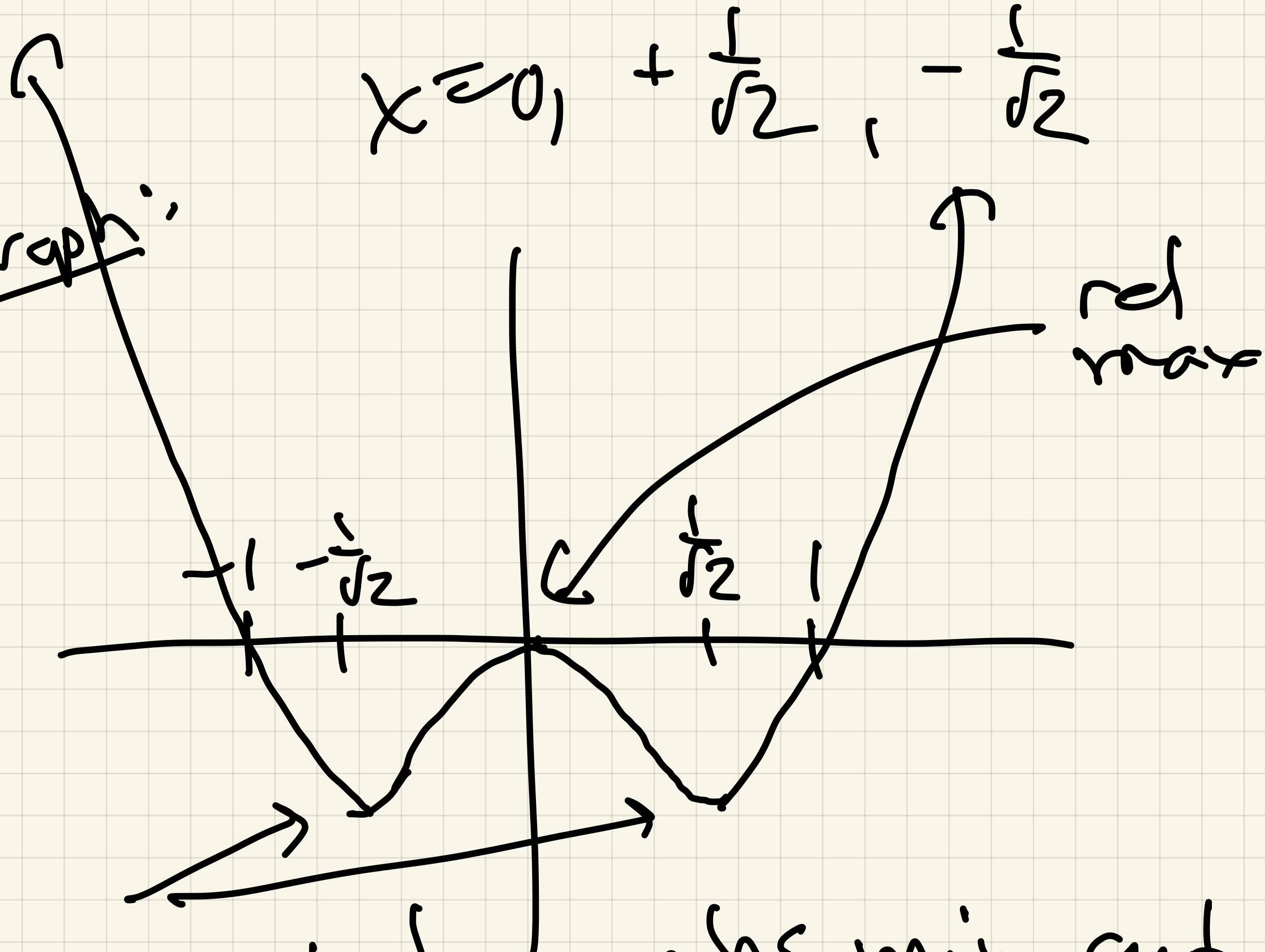
$$4x\left(x^2 - \frac{1}{2}\right) = 0$$

$$4x\left(x - \frac{1}{\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right) = 0$$

$$\Downarrow$$

$$x = 0, \ +\frac{1}{\sqrt{2}}, \ -\frac{1}{\sqrt{2}}$$

<u>Graph:</u>



$x = \pm\frac{1}{\sqrt{2}}$ abs min and rel min

The abs min value is $f(\pm\frac{1}{\sqrt{2}}) = -\frac{1}{4}$

Ex 7 Same for $f(x) = x^{2/3} - x^{5/3}$ (domain $\mathbb{R}$)

$\cancel{f'}$

$$f' = \boxed{\frac{2}{3} x^{-\frac{1}{3}}} - \frac{5}{3} x^{\frac{2}{3}}$$

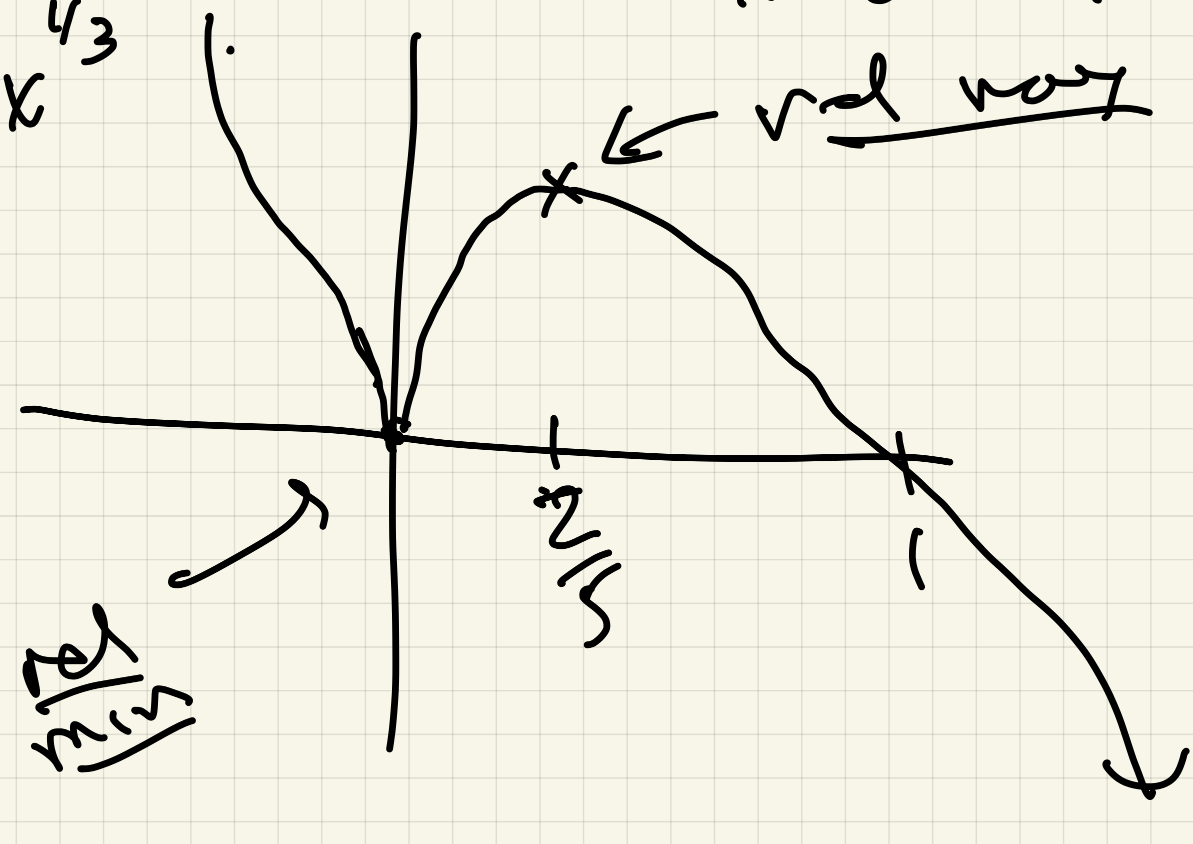
$$= \frac{1}{3} x^{-\frac{1}{3}} \left(2 - 5\boxed{x} \right)$$

crit points:

$x = 2/5 \quad f' = 0$

$x = 0 \quad f' \text{ DNE}$

$\frac{1}{3x^{4/3}}$



rel min at $x=0$ $(f=0)$

rel max at $x=2/5$ $(f(\frac{2}{5}))$