3/7 Calc I : Quiz 12

avg 94%
med 99%

1. $y = \ln(\underline{x^5 + 8x})$

$$y' = \frac{5x^4+8}{x^5+8x}$$

2. $$y = \ln \frac{(2x+1)^3}{\sin^5 x} =$$

$$= \ln(2x+1)^3 - \ln(\sin x)^5$$

$$= 3\ln(\underline{2x+1}) - 5\ln(\underline{\sin x})$$

$$3\,\frac{2}{2x+1} - 5\,\frac{\cos x}{\sin x}$$

$$= \frac{6}{2x+1} - 5\cot x$$

3. $y = 3\boxed{(\arctan x)}^5$

$$15(\arctan x)^4 \cdot \frac{1}{1+x^2}$$

4. $y = \arcsin(\underline{x^3+1})$

$$\frac{d}{dx}(\arcsin) = \frac{1}{\sqrt{1-\underline{x}^2}}$$

chain

$$y' = \frac{1}{\sqrt{1-(x^3+1)^2}} \cdot 3x^2$$

5. $\frac{d}{dx}(\operatorname{arcsec} x) = \frac{1}{|x|\sqrt{x^2-1}}$

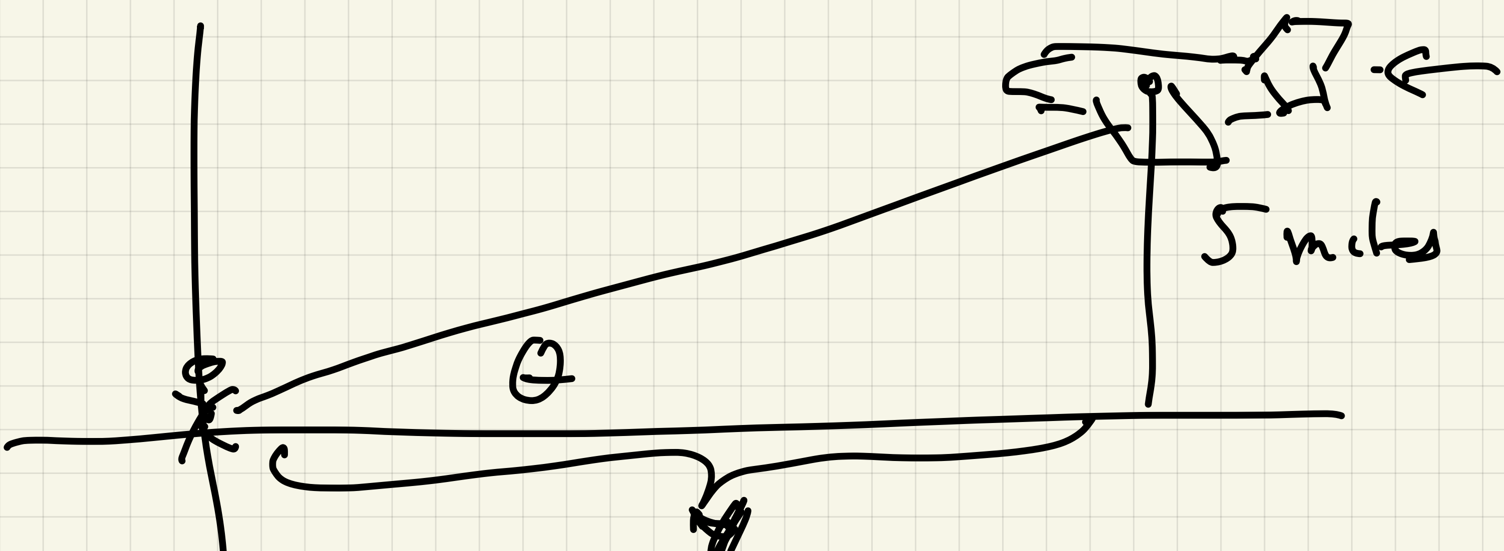
$$y = \operatorname{arcsec}\left(\frac{7x+1}{4}\right)$$

$$\frac{1}{|7x+1|\sqrt{(7x+1)^2-1}} \cdot 7$$

Last time: Related Rates

Ex 1 An observer watches an airplane approach at an altitude of 5 miles,

θ = angle of elevation



(a) If plane flies at 600 mph, what is the rate of change of θ when $\theta = 60° = \pi/3$ rad

(b) If $\frac{d\theta}{dt} = 90$ rad/hr when $\theta = 45° = \pi/4$ rads, how fast is airplane flying?

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t = time

θ = angle

y = horizontal distance to airplane

(a) $\theta = \pi/3$, $\frac{dy}{dt} = -600$ mph

$$\tan\theta = \frac{5}{y}$$

could do $\qquad \sec^2\theta \frac{d\theta}{dt} = \frac{-5\frac{dy}{dt}}{y^2}$

This is OK : but

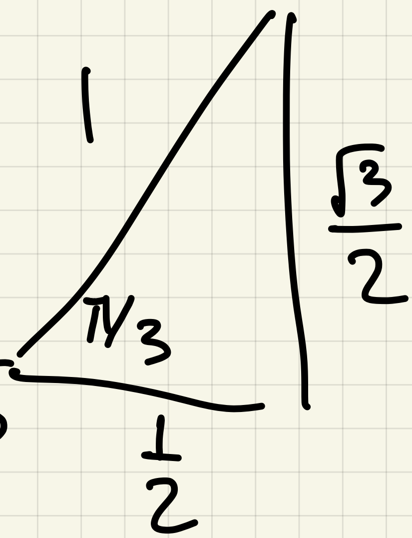
$$\cot\theta = \frac{y}{5}$$

$$\boxed{-\csc^2\theta \boxed{\frac{d\theta}{dt}} = \frac{1}{5}\frac{dy}{dt}}$$

$$-600$$

$$\csc^2\frac{\pi}{3} = \frac{1}{(\sin \pi/3)^2}$$

$$= \frac{1}{(\sqrt{3}/2)^2} = \frac{1}{(3/4)} = \frac{4}{3}$$



$$-\frac{4}{3}\frac{d\theta}{dt} = \frac{1}{5}(-600) =$$

$$\frac{d\theta}{dt} = 120\left(3/4\right) = \underline{90 \text{ rad/hr}}$$

(b) $-5\boxed{\csc^2\theta}\frac{d\theta}{dt} = \frac{dy}{dt}$

↑ ??

-5

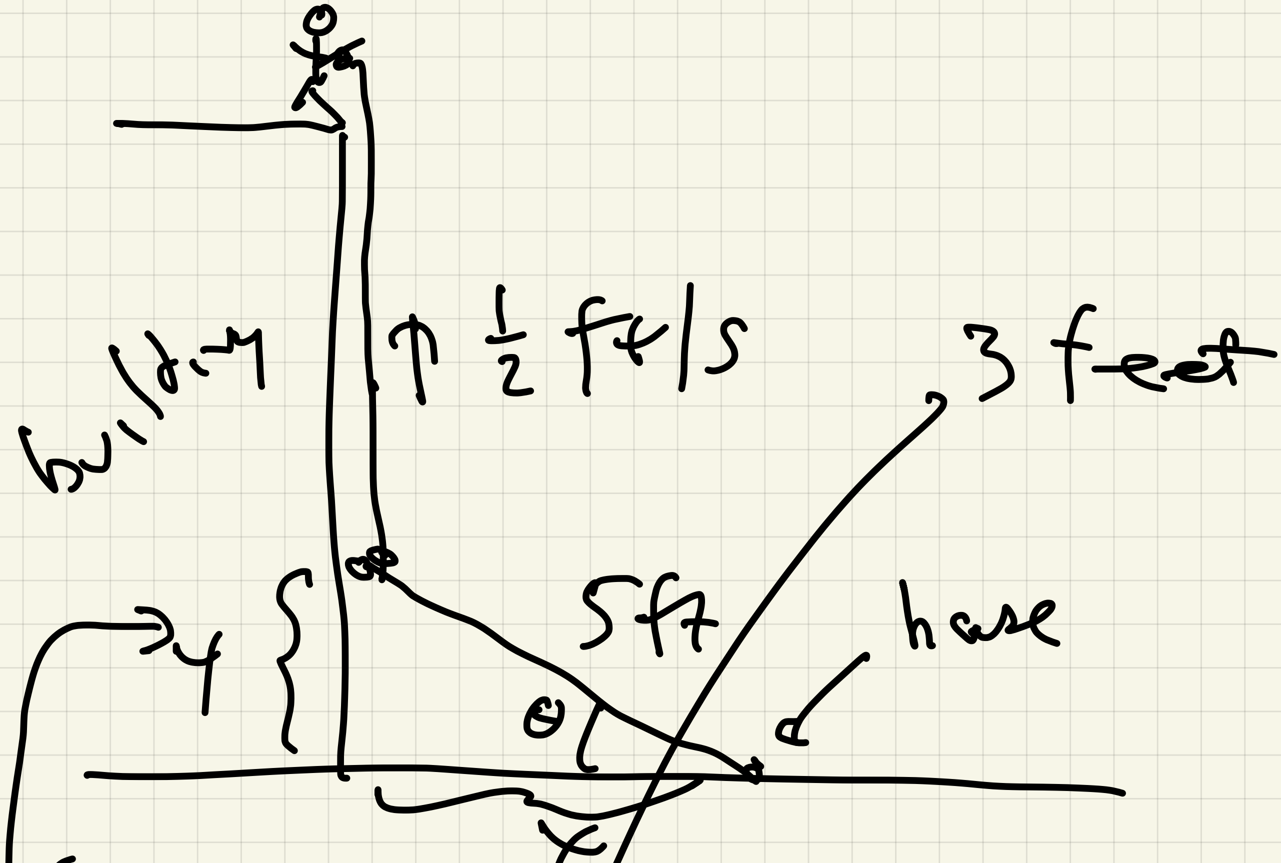
$\theta = \pi/4$, $\frac{d\theta}{dt} = 90$ rad/hr

$$\csc^2 \pi/4 = \frac{1}{\sin^2(\pi/4)} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$-5(2)\cdot 90 = \frac{dy}{dt}$$

-900 mph

Ex2: A worker hoists a 5 ft plank up side of building with a rope at $\frac{1}{2}$ ft/sec



(a) How fast is the base of plank moving when it's from bottom of building?

(b) How fast is θ changing?

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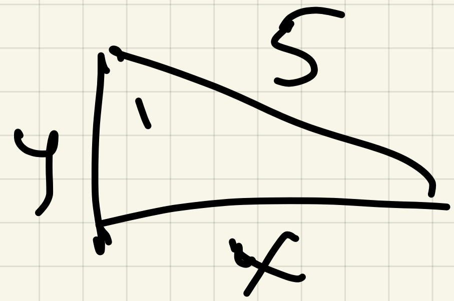
t = time
x = dist from base of plank to building
y = vert distance

θ = angle shown

$$\frac{dy}{dt} = \frac{1}{2} \text{ ft/sec} \qquad x = 3 \text{ ft.}$$

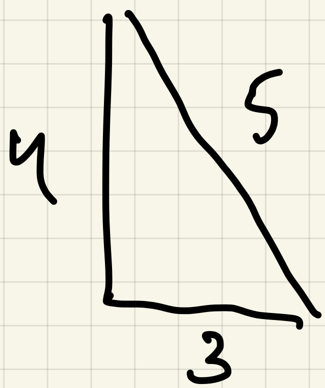
x, y related ?!



$$x^2 + y^2 = 5^2$$

d/dt

$$2\underbrace{\boxed{x}}_{=3}\boxed{\frac{dx}{dt}} + 2\underbrace{\boxed{y}}_{y=4}\underbrace{\boxed{\frac{dy}{dt}}}_{\frac{1}{2}} = 0$$



$$2 \cdot 3 \cdot \frac{dx}{dt} = -2 \cdot 4 \cdot \left(\frac{1}{2}\right)$$

$$\frac{dx}{dt} = \frac{-\cancel{2} \cdot 4 \cdot \frac{1}{2}}{\cancel{2} \cdot 3} = -2/3 \text{ ft/sec} \checkmark$$

(b) $\cos\theta = \frac{x}{5}$ ✓

$$\theta = \arccos\left(\frac{x}{5}\right)$$

$$\frac{d\theta}{dt} = \frac{-1}{\sqrt{1-\left(\frac{x}{5}\right)^2}} \cdot \frac{1}{5} \cdot \frac{dx}{dt}$$

$$= \frac{-1}{5\sqrt{1-\left(\frac{x}{5}\right)^2}} \frac{dx}{dt}$$

$$= \frac{-1}{\sqrt{25-x^2}} \frac{dx}{dt}$$

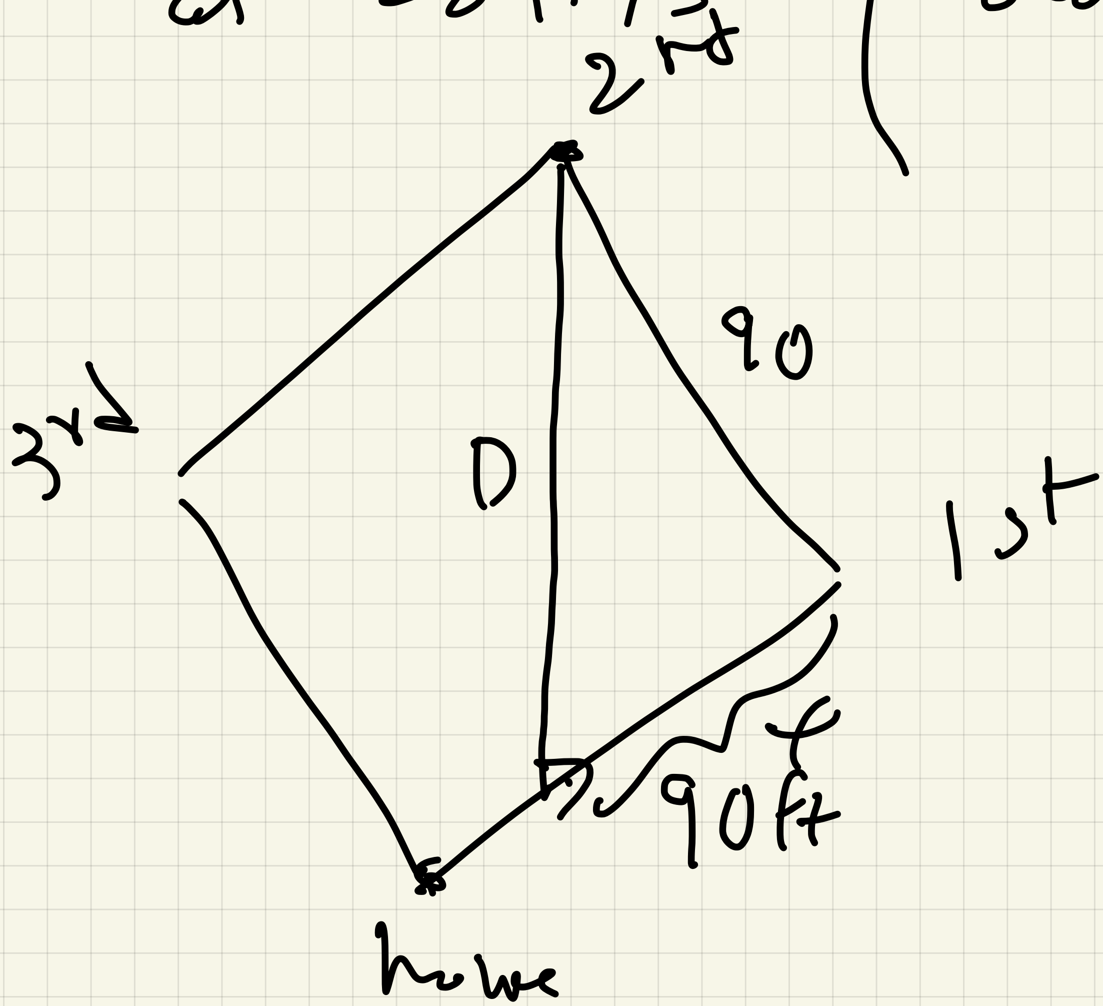
$-2/3$

$$= \frac{-1}{\sqrt{25-3^2}} \left(-2/3\right) =$$

$$-\frac{1}{4}\left(-\frac{2}{3}\right) = +\frac{1}{6}$$

rad/sec

Ex 3 A baseball player runs from home plate to 1^{st} base at $20\ ft/s$ (slow for MLB)



How fast is distance from runner to 2^{nd} base changing when the runner is $2/3$ way to 1^{st} base?

t = time

x = dist from runner to 1^{st} base

D = dist from runner to 2^{nd}

Known $\frac{dx}{dt} = -20$ ft/sec $\quad x = 30$ ft

relationship:

$\boxed{\text{Want } \frac{dD}{dt}}$ $\quad x^2 + 90^2 = D^2$

$D = \sqrt{x^2+90^2}$

$$\frac{dD}{dt} = \cancel{\frac{1}{2}}\left(\underline{\underline{x^2+90^2}}\right)^{-\frac{1}{2}} \cdot \left(\cancel{2}x^1 \cdot \frac{dx}{dt}\right)$$

$$= \frac{x \cdot \frac{dx}{dt}}{\sqrt{x^2+90^2}}$$

$x = 30$

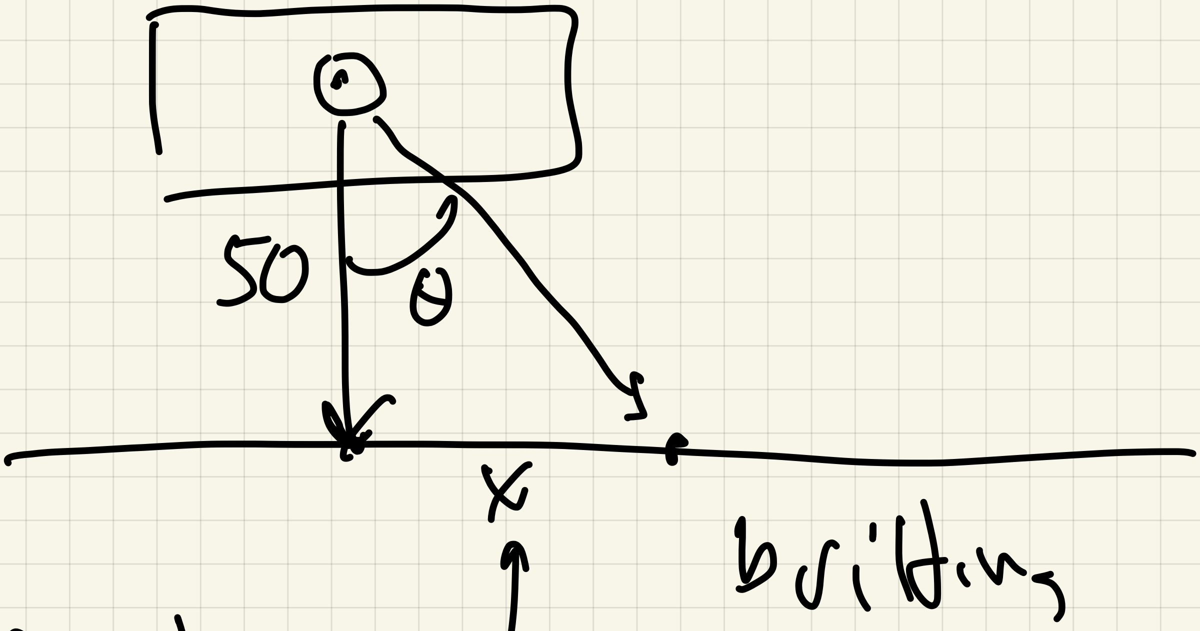
$x' = -20$

$$\frac{30(-20)}{\sqrt{\underbrace{30^2+90^2}_{(30\cdot 3)^2}}} = \frac{\cancel{30}(-20)}{\cancel{30}\sqrt{1+3^2}}$$

$$\frac{-20}{\sqrt{10}} = -2\sqrt{10} \text{ ft/sec.}$$

<u>Ex 4</u> A light rotates on top of a cop car at $\boxed{30 \text{ rev/min}}$

The beam of light moves along a wall of building 50 ft away from car. How fast is light beam moving along wall when $\theta = 0$ and $\theta = 45^\circ = \pi/4$ rad



Variables x, θ, t as shown.

Know $\frac{d\theta}{dt} = 30$ rev/min

$30 \cdot 2\pi = 60\pi$ rad/min

$$\boxed{\text{Want } \frac{dx}{dt}}$$

$$\tan\theta = \frac{x}{50}$$

$$x = 50 \tan\theta$$

$$\frac{dx}{dt} = 50 \sec^2\theta \cdot \frac{d\theta}{dt}$$

$$= 50 \frac{d\theta}{dt}$$

$$\overline{\qquad\qquad}$$
$$\cos^2\theta$$

$\underline{\theta = 0}$

$$\frac{dx}{dt} = 50\cdot(60\pi) =$$

$\cos 0 = 1$

$$3000\pi \text{ ft/min} = 107.1 \text{ mph}$$

$\theta = 45° = \left(\pi/4\right)$ rad

$$\cos 45 = \frac{1}{\sqrt{2}}$$

$$\frac{dx}{dt} = \frac{50\,\frac{d\theta}{dt}}{\cos^2 45} = \frac{50\,\frac{d\theta}{dt}}{\left(\frac{1}{\sqrt{2}}\right)^2} =$$

$$\frac{50\,\theta'}{\left(\frac{1}{2}\right)} = 100\,\theta' = 6000\pi \text{ ft/min} = 214 \text{ mph}$$