3/5 | Calc I

Quiz 4

$$xy + y^3 = x^3 + 2x - 1 \qquad \frac{d}{dx}$$

$$1 \cdot y + x\frac{dy}{dx} + 3y^2 \cdot \frac{dy}{dx} = 2x + 2$$

$$(x + 3y^2)\frac{dy}{dx} = 2x + 2 - y$$

$$\frac{dy}{dx} = \frac{2x+2-y}{x+3y^2}$$

(b) at $(3,2) = (x,y)$

$$m = \frac{2(3)+2-2}{3+3\cdot 4} = \frac{6}{15} = \frac{2}{5}$$

Last time: §3.4

1-dimensional motion

$t =$ time

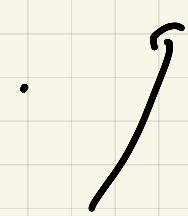
$s(t) =$ position

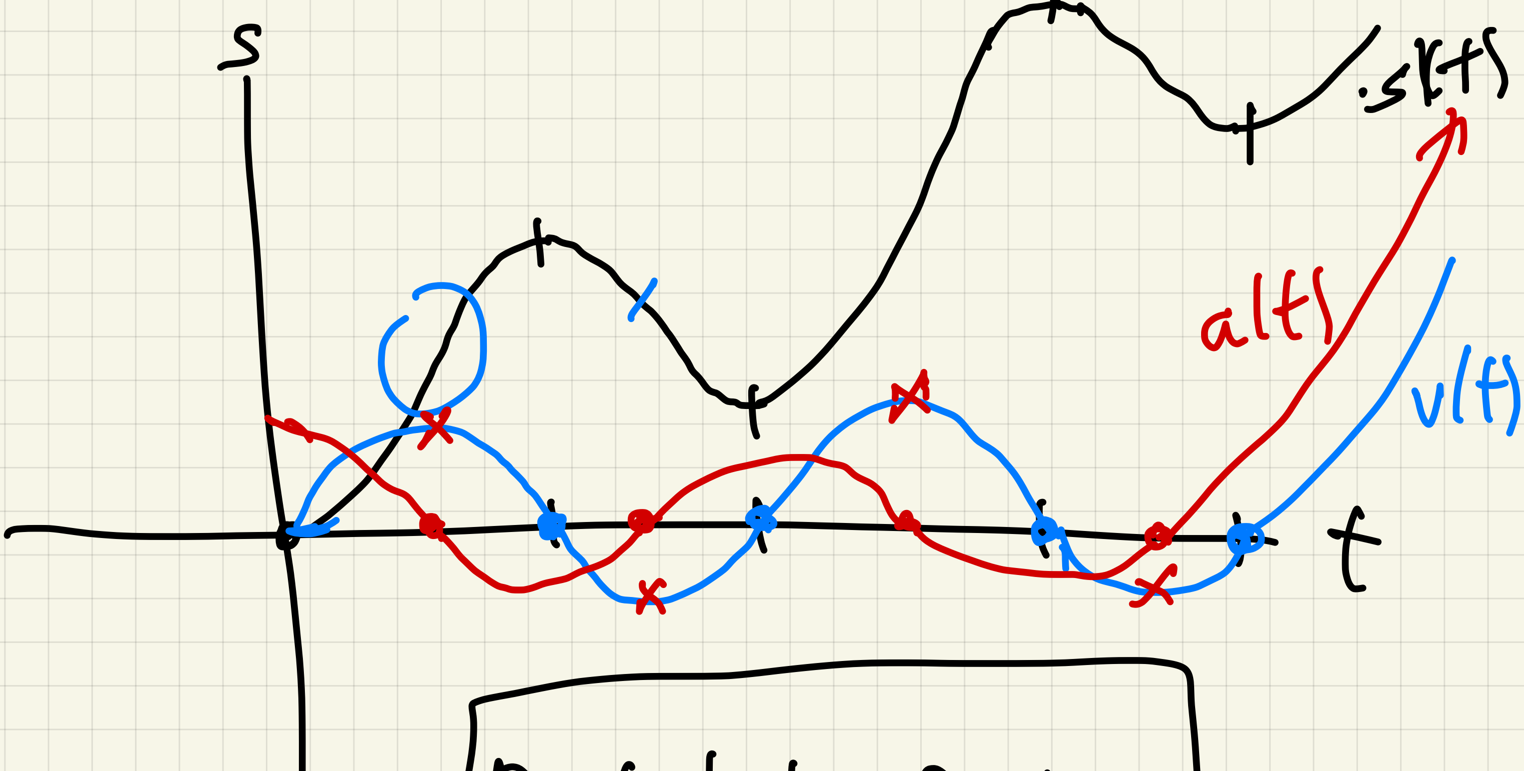
$\frac{ds}{dt} =$ velocity $\quad \left|\frac{ds}{dt}\right| =$ speed

$\frac{d^2s}{dt^2} =$ acceleration

<u>Ex</u> Graph below gives position of particle at time $t \geq 0$:

sketch graph of velocity/acceleration



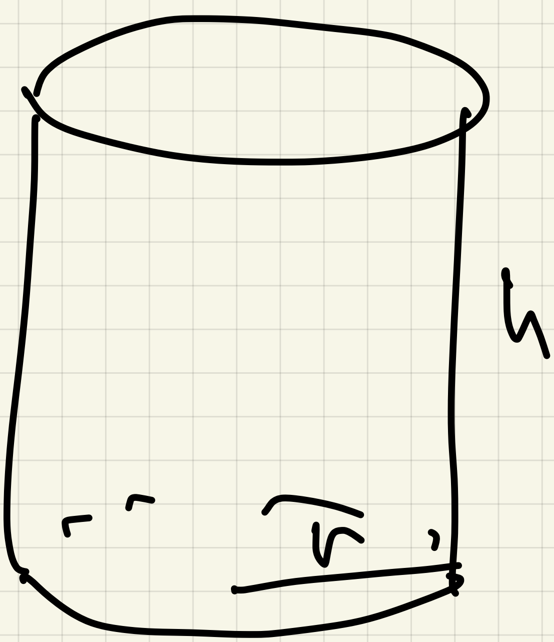


§ 3.6 $\boxed{\text{Related Rates}}$

Usually t = time

Ex : A cylindrical can of radius r and height h has

$V = \pi r^2 h$



If h, r, V change with t,
(a) how are $\frac{dh}{dt}$, $\frac{dr}{dt}$ & $\frac{dV}{dt}$ related?,

(b) If at some time $h = 10$ cm
$r = 50$ cm

$\frac{dh}{dt} = -6$ ccm/s, $\frac{dr}{dt} = 4$ cm/s

what is $\frac{dV}{dt}$??

$$V = \pi r^2 h \qquad d/dt$$

$$\frac{dV}{dt} = \pi \left[2r^1 \cdot \frac{dr}{dt} \cdot h + r^2 \cdot \frac{dh}{dt} \right]$$

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$

(b) <u>With data</u> $h = 10$ $h' = -6$

$r = 50$ $r' = 4$

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$$2\pi(50)(10)(4) + \pi\, 50^2 \cdot (-6)$$

$$4000\pi - 15000\pi$$

$$= -11000\pi \text{ cm}^3/\text{s}$$

<u>Ex2</u> A spherical balloon is inflated at $\boxed{1.5 \text{ ft}^3/\text{s}}$.

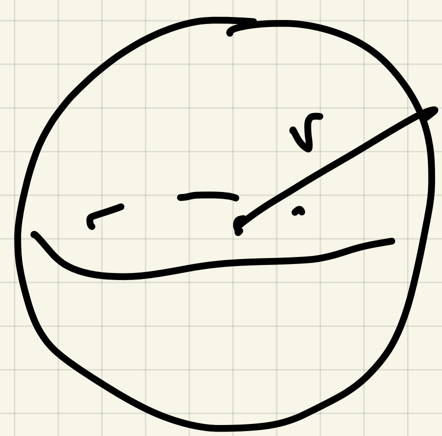
How fast is radius increasing when $\boxed{r = 2 \text{ ft}}$

<u>variables:</u>

t = time

V = volume

r = radius



Know

$$V = \frac{4}{3}\pi r^3$$

$\boxed{r = 2}$

$\boxed{\frac{dV}{dt} = 1.5 = 3/2}$

d/dt

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cancel{3} r^2 \cdot \frac{dr}{dt}$$

$$\boxed{\frac{dV}{dt}} = 4\pi \boxed{r^2} \frac{dr}{dt}$$

(Surface area)

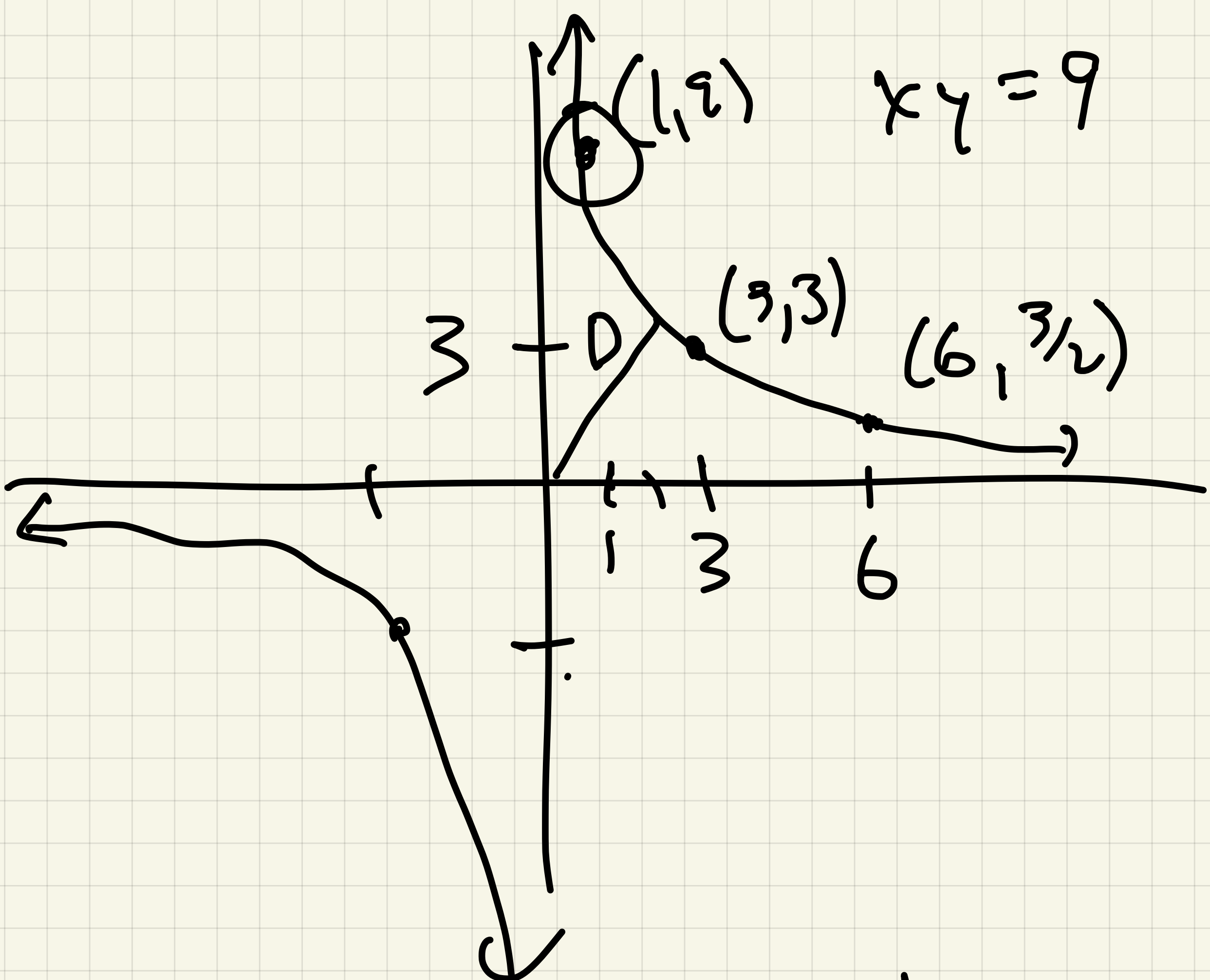
$$1.5 = \frac{3}{2} = 4 \cdot \pi \cdot 2^2 \cdot \frac{dr}{dt}$$

$$\frac{3}{2 \cdot 4 \cdot \pi \cdot 2^2} = \frac{dr}{dt}$$

$$\| $$

$$\frac{3}{32\pi} \text{ ft/s}$$

Ex3 A point $P = (x, y)$ moves in x-y plane along hyperbola $xy = 9$ in such a way that x coord moves right at 2cm/s



(a) find the velocity/speed of

y coordinate when

$x = 1, 3, 6$ cm

(b) How fast is the distance D from P to origin changing when $x = 1, 3, 6$??

(a) $\frac{dx}{dt} = 2$

t = time
x = x coord of P
y = y coord of P
D = distance of P to $(0,0)$

<u>relationship</u>: $\boxed{xy = 9}$

$\boxed{y = \frac{9}{x}}$ ← <u>diff this</u>

$\Downarrow$

$9x^{-1}$

$y = 9x^{-1}$

$$\frac{dy}{dt} = 9\cdot(-1)x^{-2}\cdot\frac{dx}{dt}$$

$$= \frac{-9\frac{dx}{dt}}{x^2} \qquad \frac{dx}{dt} = 2\,cm/s$$

(a) $\boxed{\underline{x=1} : \frac{dy}{dt} = \frac{-9\cdot 2}{1^2} = -18\,cm/s}$

$\underline{x=3} : \frac{dy}{dt} = \frac{-9\cdot 2}{3^2} = -2\,cm/s$

$\underline{x=6} : \frac{dy}{dt} = \frac{-9\cdot 2}{6^2} = -\frac{1}{2}\,cm/s$

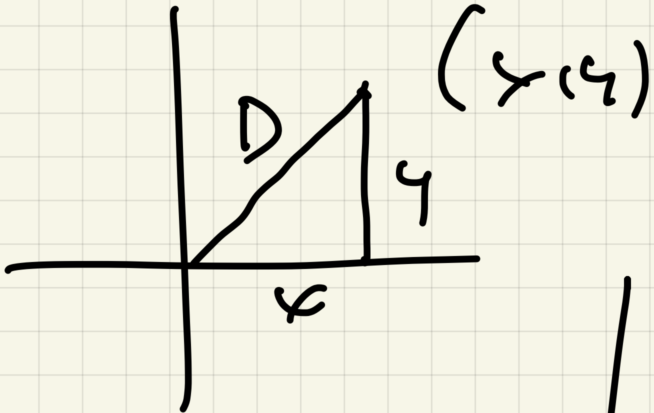
<u>Compare</u> : $xy = 9$

$$x\cdot\frac{dy}{dt} + \frac{dx}{dt}\cdot y = 0$$

$$\Downarrow$$

$$\frac{dy}{dt} = \frac{-y\frac{dx}{dt}}{x}$$

(b) D = distance ~~to~~ origin



$$\boxed{x^2+y^2 = D^2}$$

$$\Downarrow$$

$$\boxed{D = \sqrt{x^2+y^2}}$$

$$(x^2+y^2)^{1/2}$$

$$\frac{dD}{dt} = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\left[2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right]$$

$$= \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}} \qquad y = \frac{9}{x}$$

$x=1$: $\frac{dx}{dt} = x' = 2$,

$x = 1$ $\qquad \frac{dy}{dt} = y' = -18$

$y = 9$

$$\frac{dD}{dt} = \frac{1\cdot 2 + 9(-18)}{\sqrt{1^2+9^2}} = \frac{-160}{\sqrt{82}}\ \text{cm/s}$$

$\underline{x=3}$ $\quad x' = 2\ ,\ x = 3$

$y' = -2 \quad y = 3$

$$\frac{dD}{dt} = \frac{3 \cdot 2 + 3(-2)}{\sqrt{9+9}} = 0 \quad !$$

$\underline{x=6}$ $\quad x' = 2\ ,\ x = 6$

$y' = -\frac{1}{2} \quad y = 3/2$

$$\frac{dD}{dt} = \frac{12 + (-3/4)}{\sqrt{38.25}} \cong 1.819 \text{ cm/s}$$

<u>Note</u>: Can also use

$$D^2 = x^2 + y^2 \longleftarrow d/dt$$

$$2D \cdot \frac{dD}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$\frac{dD}{dt} = 2 \frac{x \, x' + y \cdot y'}{D}$$