3/28/ Calc:

§4.3 $y = f(x)$ is incr / decr

(A) $f' > 0$ / $f' < 0$

rel max/mins

§4.4 $y = f(x)$ is concave up/down

(B) f' incr / f' decr

$f'' > 0$ / $f'' < 0$

Using (A) + (B) can sketch graph of $y = f(x)$

(A) + (B), can add other information:

Intercepts

HA

VA

symmetry

Ex1 Sketch graphs satisfying: $y = f(x)$

(a) $f(2) = f(4) = 0$

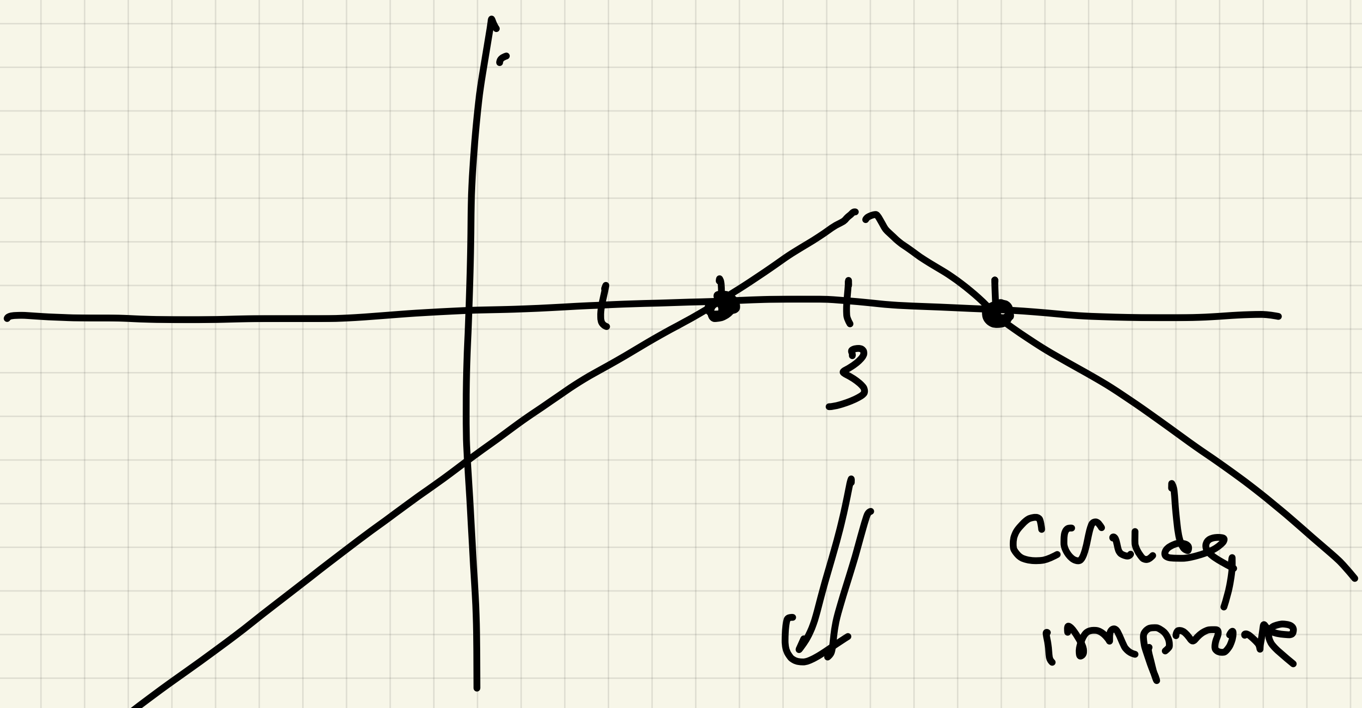
$f'(3) = 0$

$f'(x) > 0$ on $(-\infty, 3)$

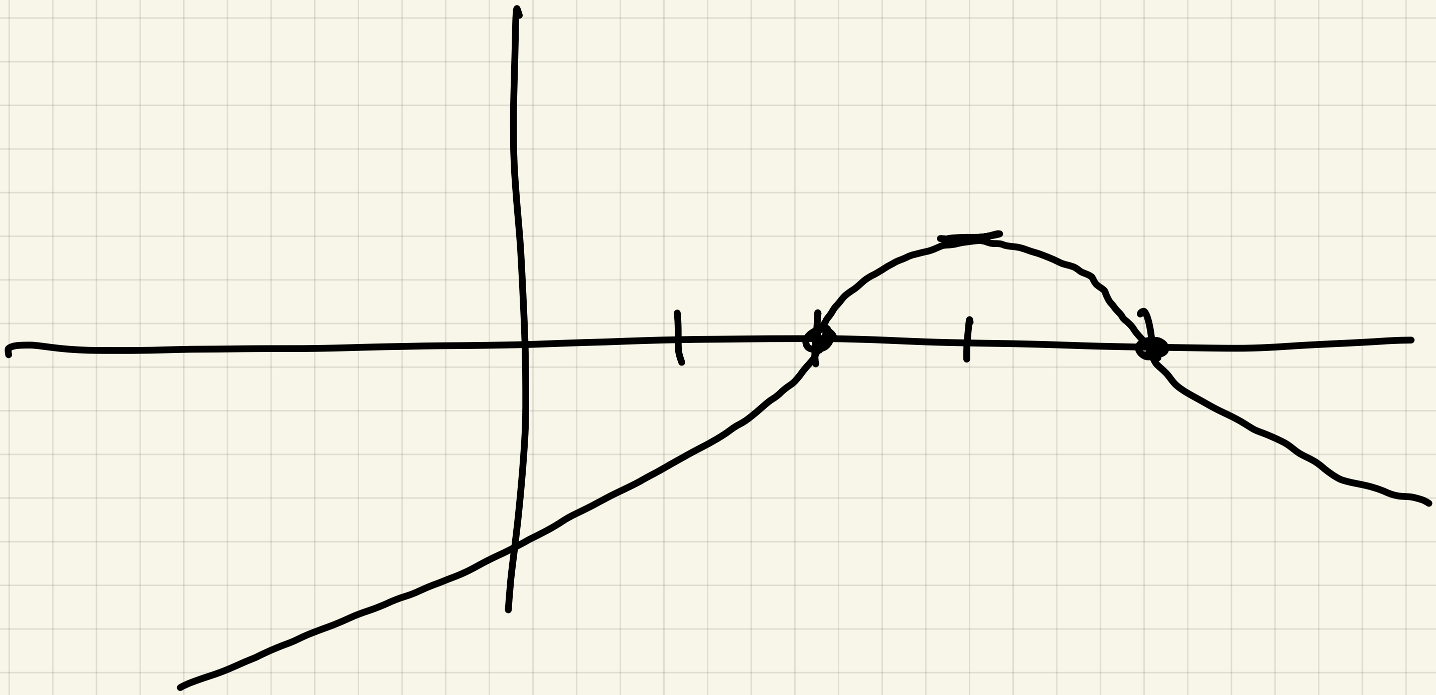
$f'(x) < 0$ on $(3, \infty)$

$f''(x) > 0$ on $(-\infty, 2) \cup (4, \infty)$

$f''(x) < 0$ on $(2, 4)$



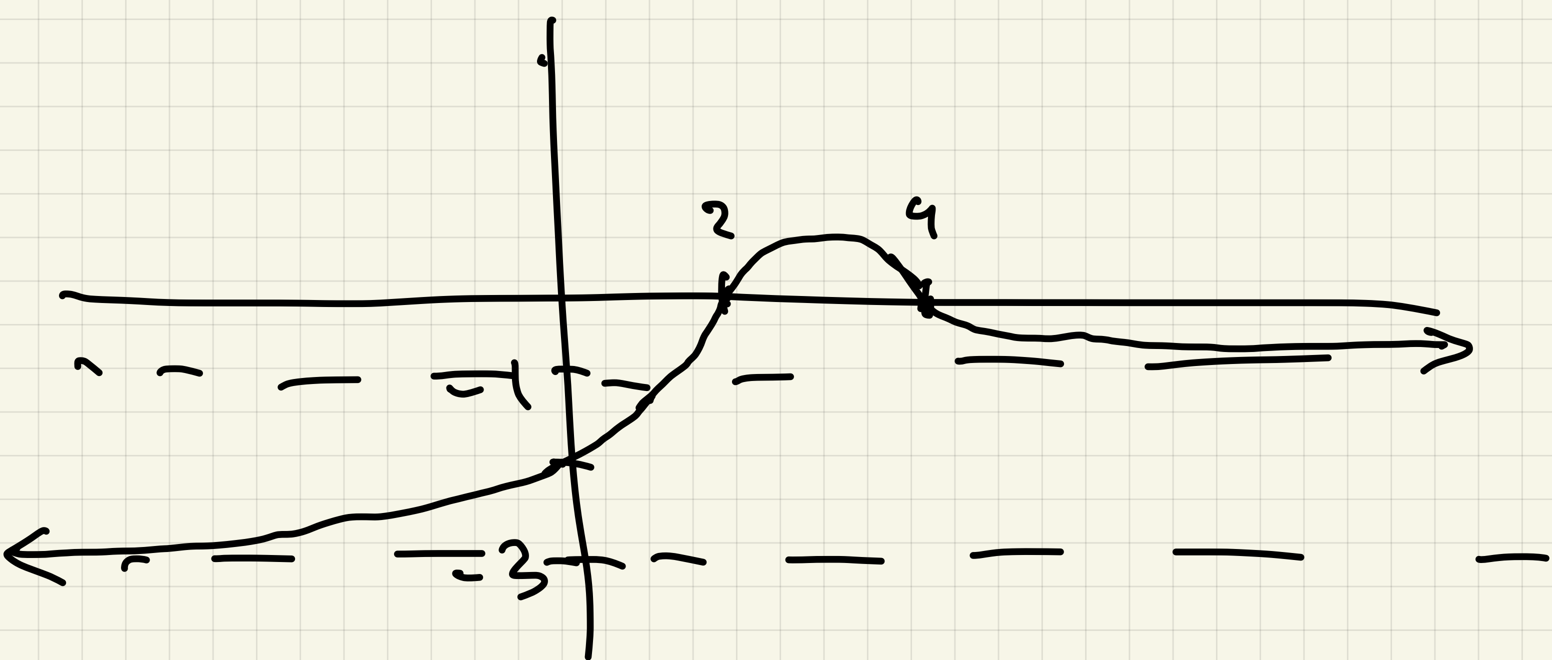
crude
improve



(a') same as (a), but also

$$\lim_{x \to \infty} f(x) = -1 \leftarrow$$

$$\lim_{x \to -\infty} f(x) = -3$$



(b) $f(x) = f(-x)$ all x

$f'(0)$ DNE

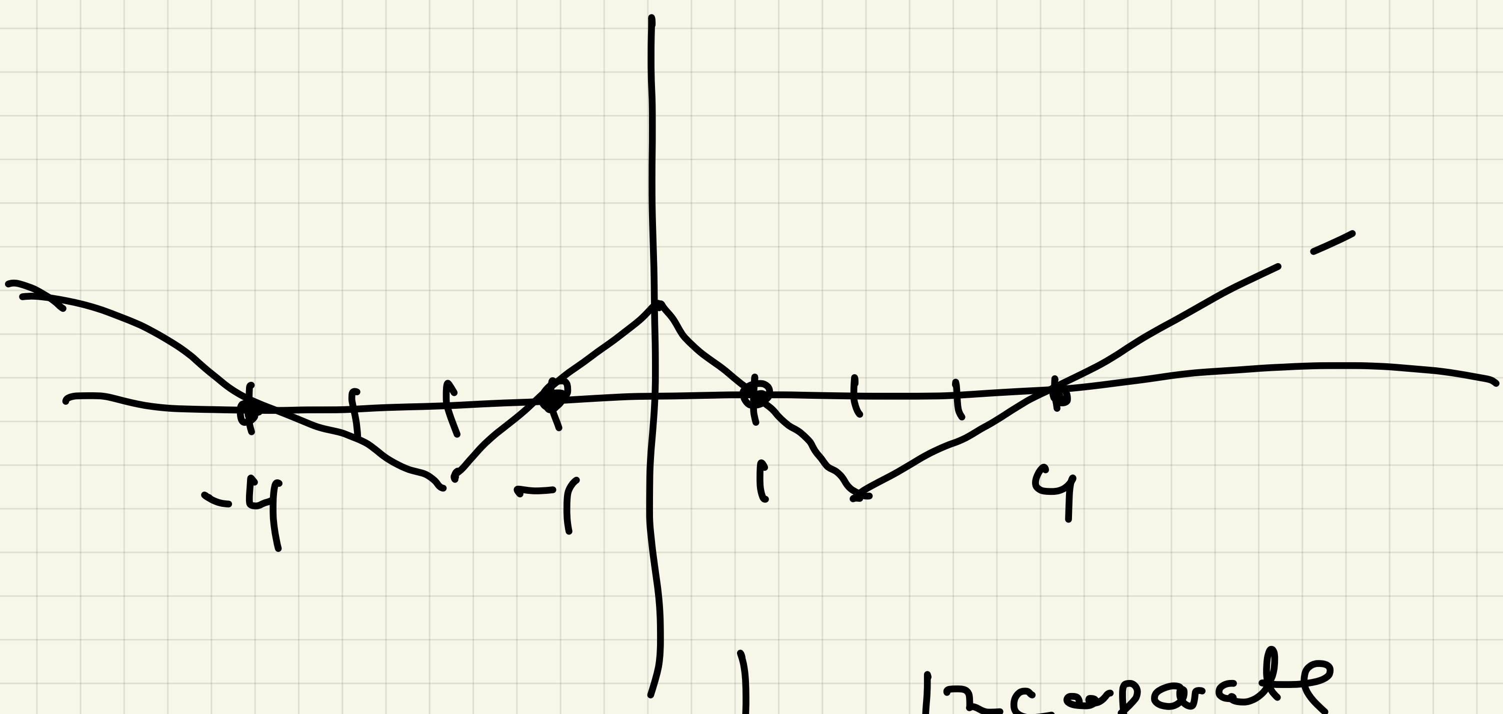
$f'(\pm 2) = 0$

$f(\pm 1) = f(\pm 4) = 0$

$f''(x) > 0$ all $x \neq 0$

$f'(x) > 0$ on $(-2,0) \cup (2,\infty)$

$f'(x) < 0$ on $(-\infty,-2) \cup (0,2)$



Incorporate $f'' > 0$

(c) $f(x) \geq 0$ all x

$f(-2) = f(4) = 0$

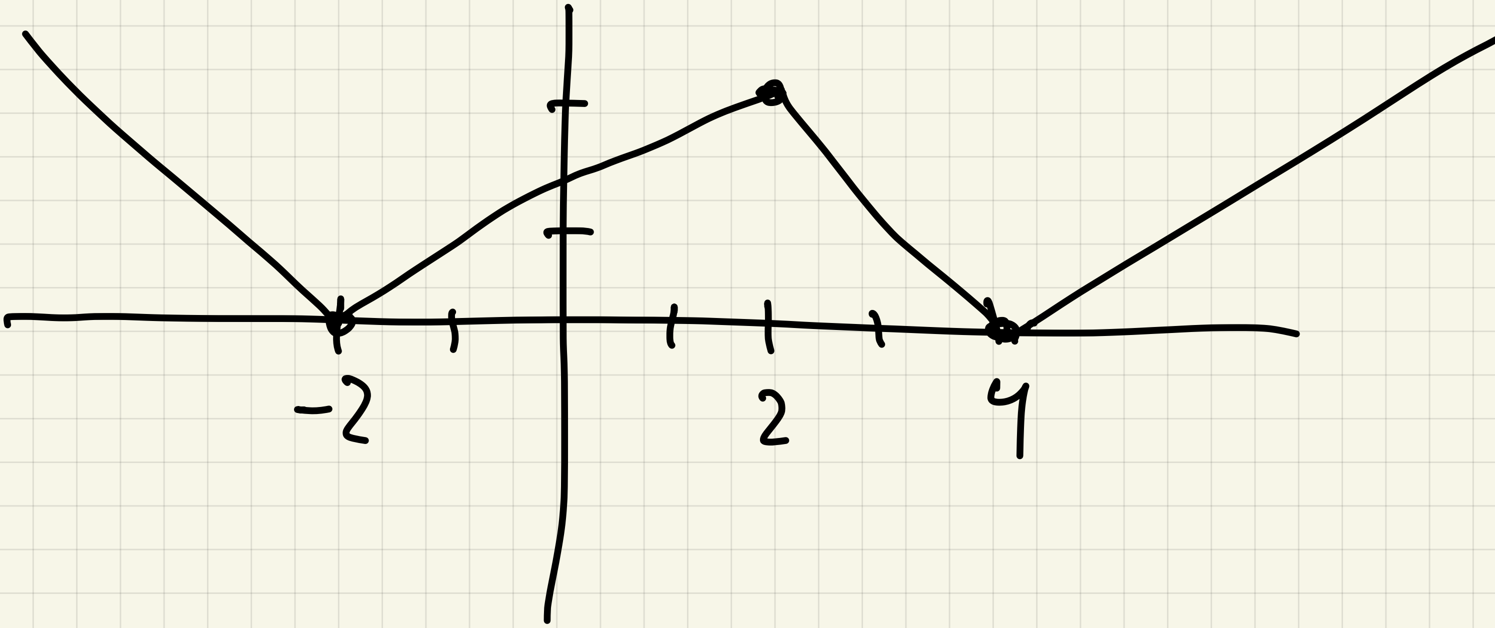
$f(2) = 2$

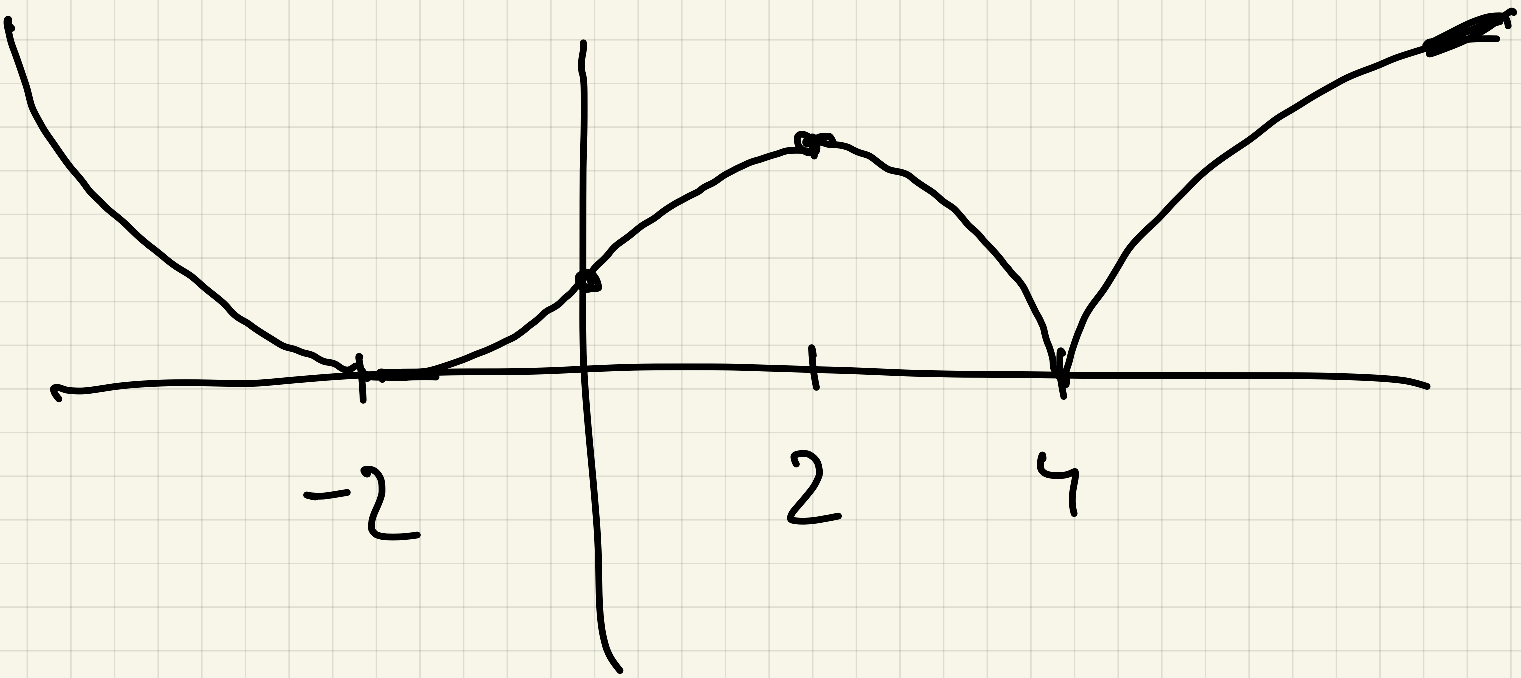
$f'(x) > 0$ on $(-2,2) \cup (4,\infty)$

$f'(x) < 0$ on $(-\infty,-2) \cup (2,4)$

$f'' > 0$ $(-\infty, 0)$

$f''(x) < 0$ on $(0,4) \cup (4,\infty)$





Ex 2 Sketch graph of

$$y = \frac{8x^2}{1+x^2}$$

Graph using A B C

Ⓐ $$y' = \frac{(1+x^2)(16x) - 2x(8x^2)}{(1+x^2)^2}$$

$$= \frac{16x + \cancel{16x^3} - \cancel{16x^3}}{(\quad)^2} =$$

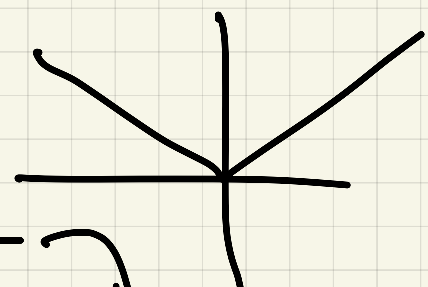
$$y' = \frac{16x}{(1+x^2)^2}$$

$x = 0$
crit pt

0

$y' < 0$ $\quad$ $y' > 0$

rel min



Ⓑ

$$y'' = \frac{(1+x^2)^2 \cdot 16 - 2(1+x^2)^1 \cdot 2x \cdot 16x}{(1+x^2)^4}$$

$\div (1+x^2)$

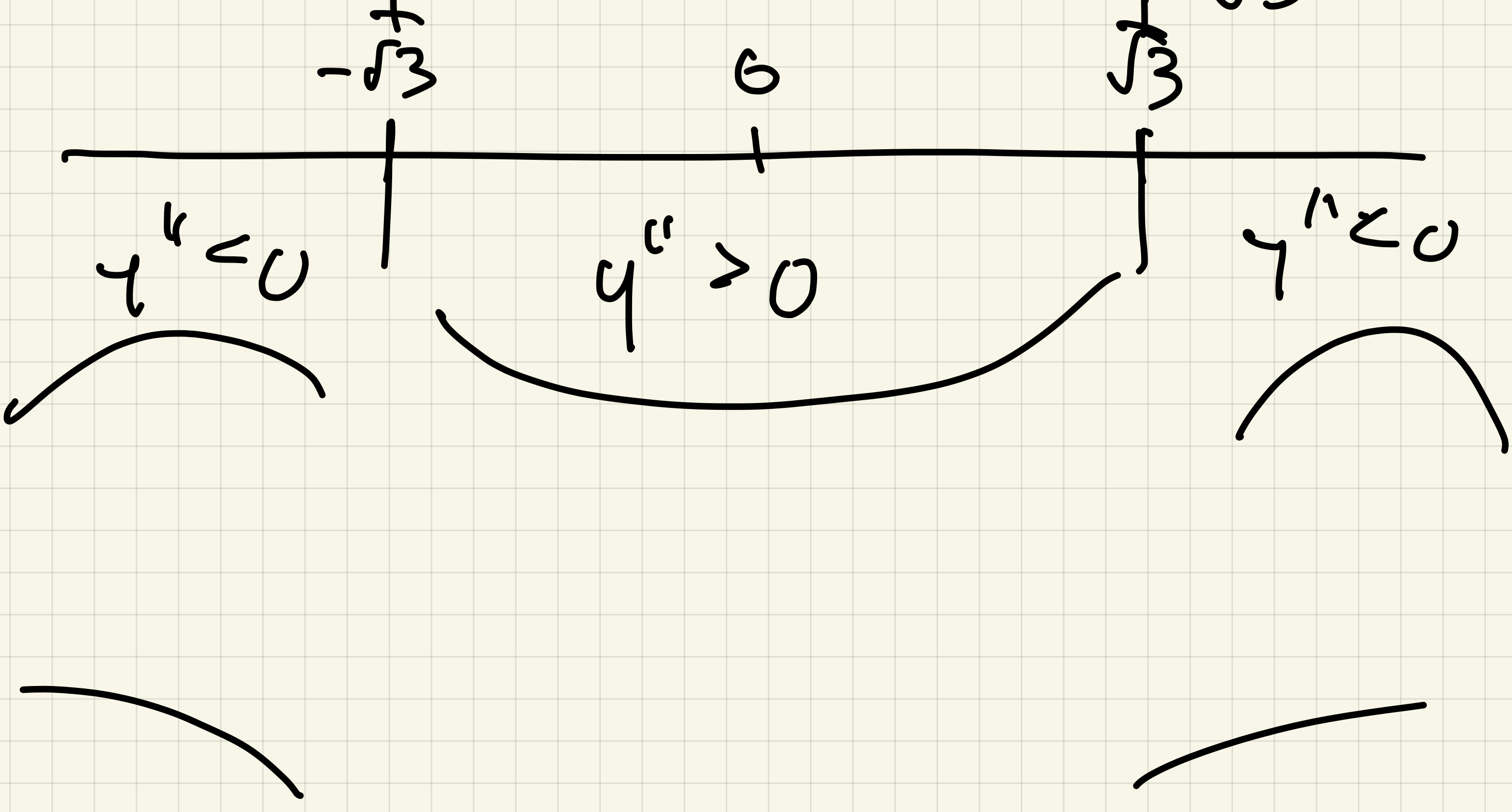
$$= \frac{(1+x^2)^1 \cdot 16 - 2 \cdot 2x \cdot 16x}{(1+x^2)^3}$$

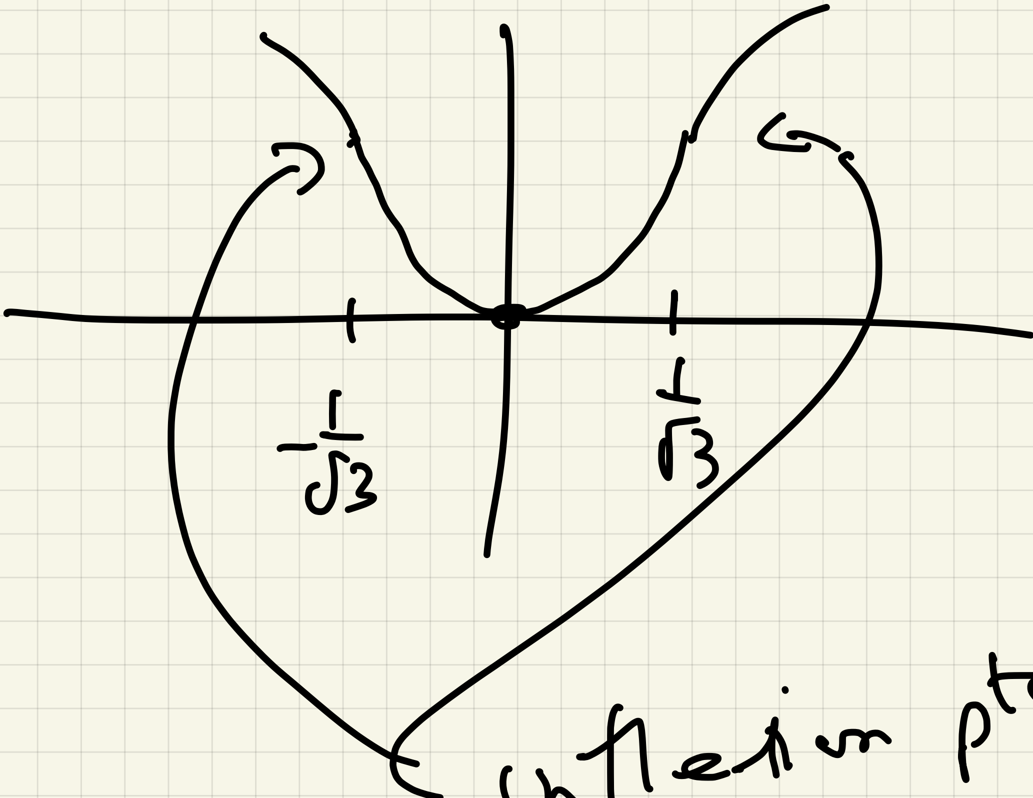
$$= \frac{16 + 16x^2 - 64x^2}{(1+x^2)^3}$$

$$= \frac{16 - 48x^2}{(1+x^2)^3} = \boxed{\frac{16(1-3x^2)}{(1+x^2)^3}}$$

$= 0$

at $x = \pm \frac{1}{\sqrt{3}}$





inflexion pts

$\left(\pm\frac{1}{\sqrt{3}}, 2\right)$ ✓

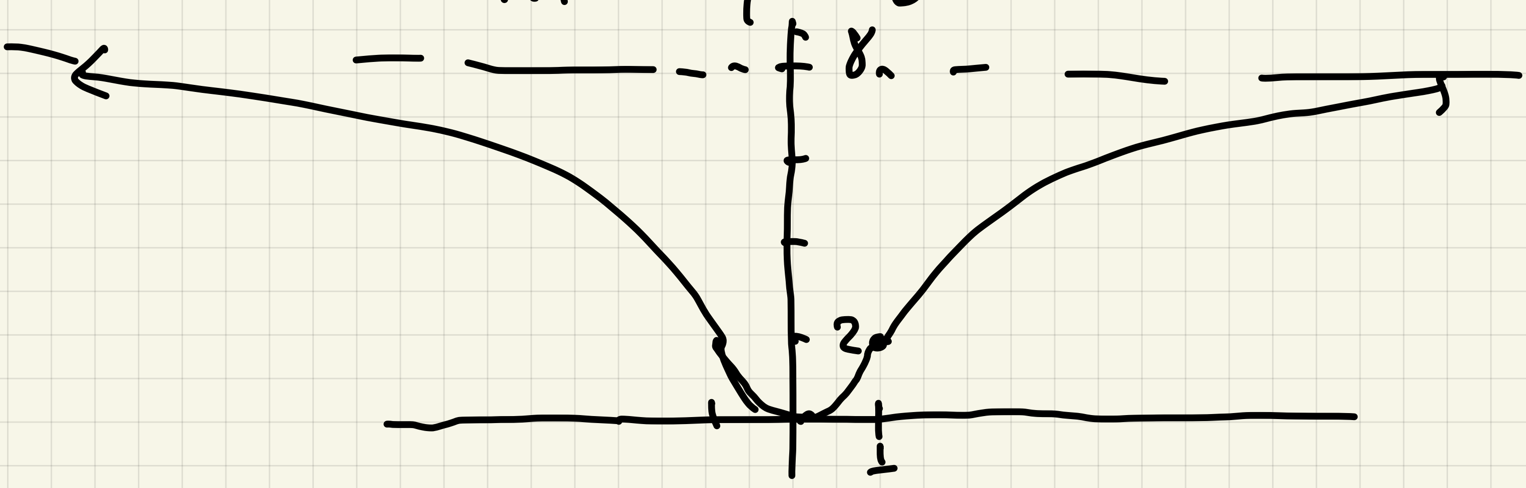
ⓒ

~~A~~ symmetry

$$\lim_{x\to\infty} \frac{8x^2}{1+x^2} = \lim_{x\to\infty} \frac{8}{\frac{1}{x^2}+1} = 8$$

$\frac{1}{x^2} \to 0$

HA : $y = 8$



$-\frac{1}{\sqrt{3}}$ $\sqrt{3}$

Ex3 $y = \frac{x^2-25}{(x^2-16)\cancel{(x+5)}} = (x-5)\cancel{(x+5)}$

Domain: $x \neq \pm 4, -5$

$x \neq -5$

$$y = \frac{x-5}{(x^2-16)}$$

y_1

y int: $\frac{-5}{-16} = \frac{5}{16} = .3125$

x int: $x = 5$

VA: $x = \pm 4$

HA $y = 0$

$$\frac{dy}{dx} = \frac{(x^2-16)(1) - (x-5)(2x)}{(x^2-16)^2}$$

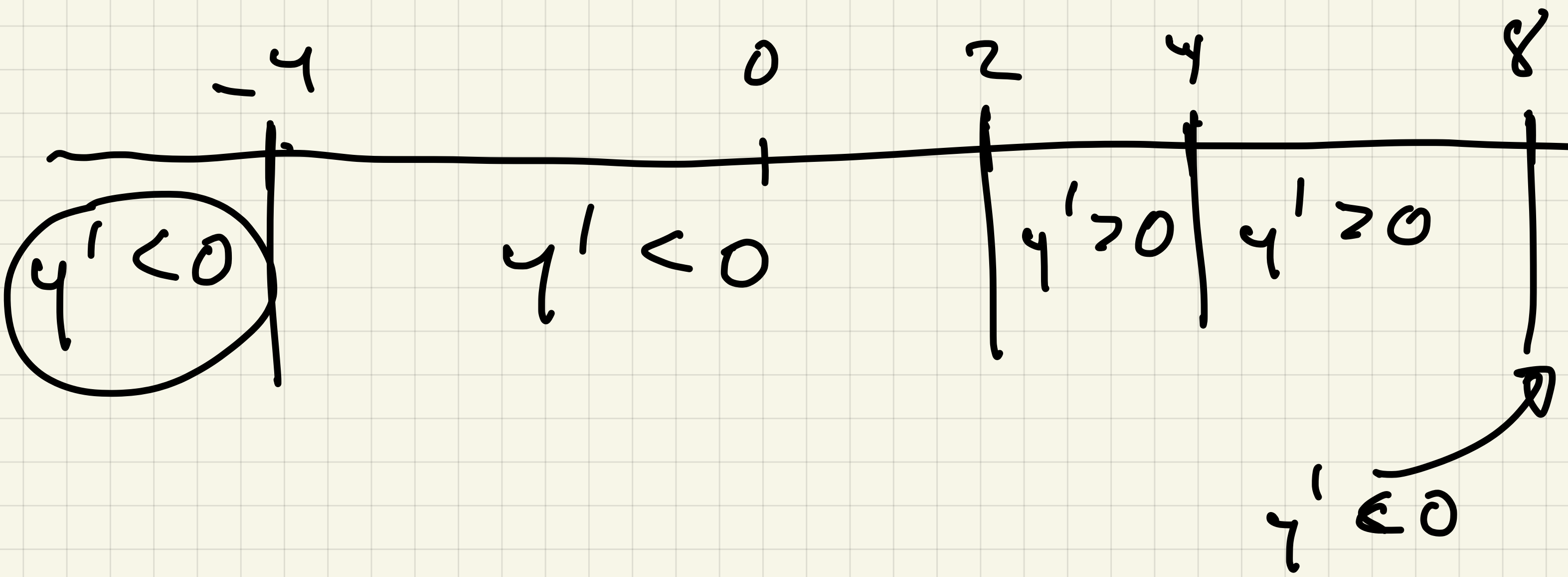
$$= \frac{x^2 - 16 - 2x^2 + 10x}{(\quad)^2}$$

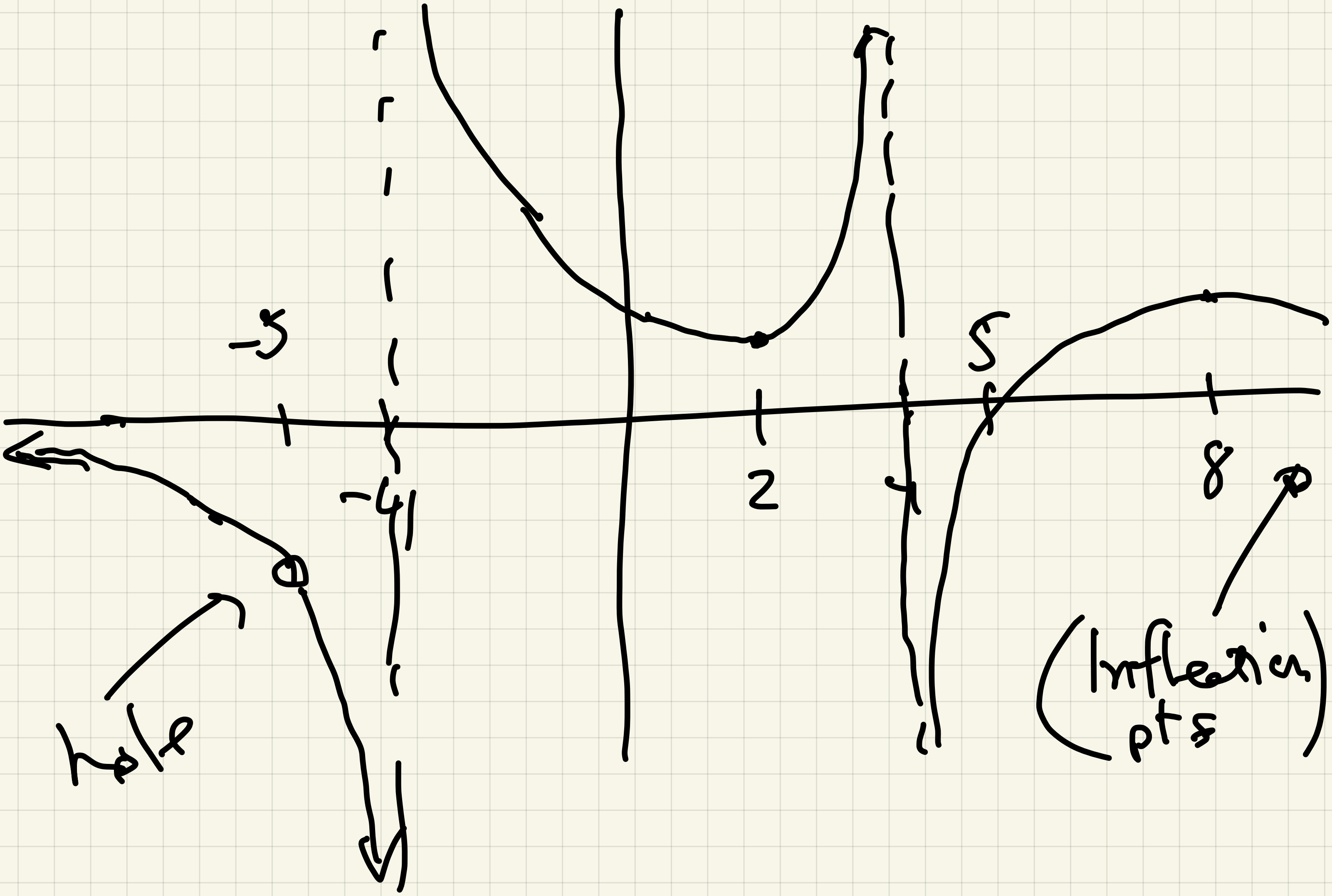
$$= \frac{-x^2+10x-16}{(x^2-16)^2}$$

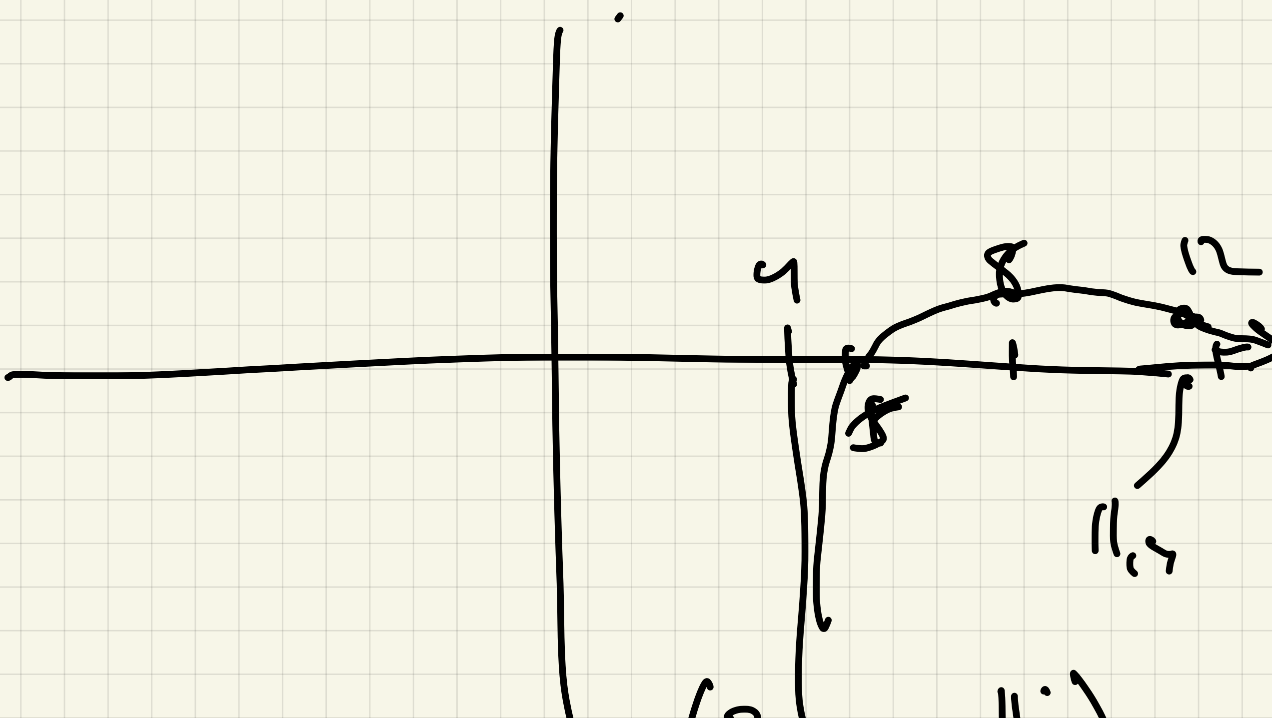
$$= \frac{-(x^2-10x+16)}{(x^2-16)^2}$$

$$\frac{-(x-2)(x-8)}{(x^2-16)^2} = 0$$

at $x = 2, 8$, DNE at $x = \pm 4$







(Bernoulli)

$\boxed{\S 4.5}$ L'Hospital's rule

$$\lim_{x\to c}\frac{f(x)}{g(x)} \cancel{=} \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

If $f(c) = 0$

$g(c) = 0$

If limit exists

Ex1 (a)

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4} = \lim_{x\to 2}\frac{3x^2}{2x} = \frac{12}{4} = 3 \checkmark$$

Ch2

$$\lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}$$

$$= \lim_{x\to 2}\frac{x^2+2x+4}{x+2} = \frac{12}{4} = 3$$