3/26/Calc: Quiz 14

1. $f(x) = x^{2/3} + x^{5/3}$

$$f' = \boxed{\frac{2}{3}x^{-1/3}} + \frac{5}{3}x^{2/3}$$

$$= \frac{1}{3}x^{-1/3}(2+5x)$$

Crit pts:

$x = 0$

$2+5x=0$

$\frac{1}{\sqrt[3]{x}}$, $x = -\frac{2}{5}$

$2=-5x$

$-\frac{2}{5}=x$

2. $f(x) = x^3+3x^2$ on $[-4,1]$

$$f'(x) = 3x^2+6x = 0$$

$$3x(x+2) = 0$$

$$x = 0, -2$$

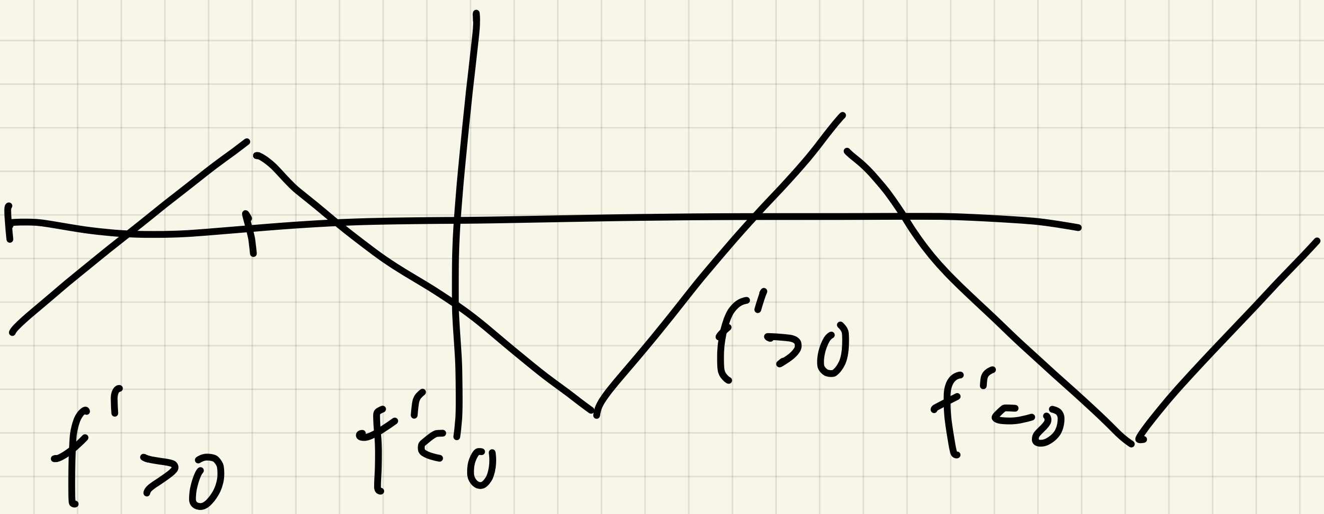
crit pts $\begin{cases} f(0) = 0 \\ f(-2) = 4 \end{cases}$ abs min

end pts $\begin{cases} f(-4) = -16 \\ f(1) = 4 \end{cases}$ abs max

$-64 + 12$
48

§4.4 <u>Concavity and curve sketching</u>

§4.3 tells us the contour of graph of $f(x)$



<u>Defn</u>: $y = f(x)$ differentiable on interval I.

(a) $f(x)$ is concave up on I

if $f'(x)$ increases on I

(b) $f(x)$ is concave down on I

if $f'(x)$ decreases

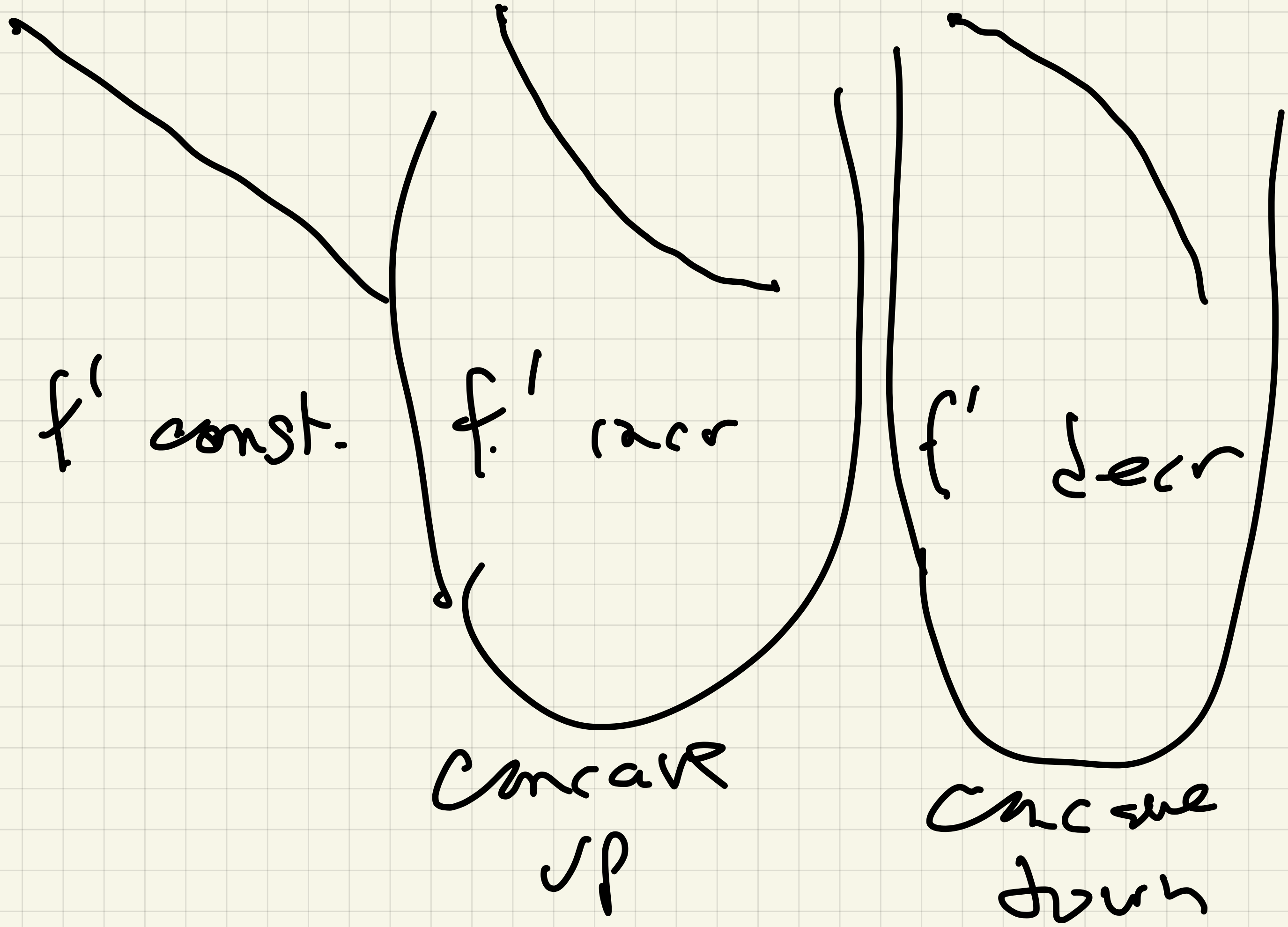
NOTICE: f' increases

$$\Updownarrow$$

$$f'' > 0$$

$$f' \text{ decreases} \iff f'' < 0$$

Graphically:



Ex $y = f(x) = 3x^4 - 4x^3$

Find open intervals where f is increas / decreas and where f is concave up/down

① $\frac{dy}{dx} = 12x^3 - 12x^2$

$= 12x^2(x-1) = 0$

at $x = 0, 1$

$0 \qquad\qquad 1$

$f'(-1) = -24 < 0$ $f' < 0$ $f'(2) = 48 > 0$

$f' < 0$ $f'(\frac{1}{2}) = -\frac{3}{2}$ $f' > 0$

f incr on $(1, \infty)$

f decr on $(-\infty, 1)$

(2) $\frac{d^2y}{dx^2} = 36x^2 - 24x$

$= \underline{\underline{12x(3x-2)}} = 0$

at $x = 0, 2/3$

$0 \qquad\qquad 2/3$

$f''(-1) > 0$ $f''(\frac{1}{3}) < 0$ $f''(1) = 12 > 0$

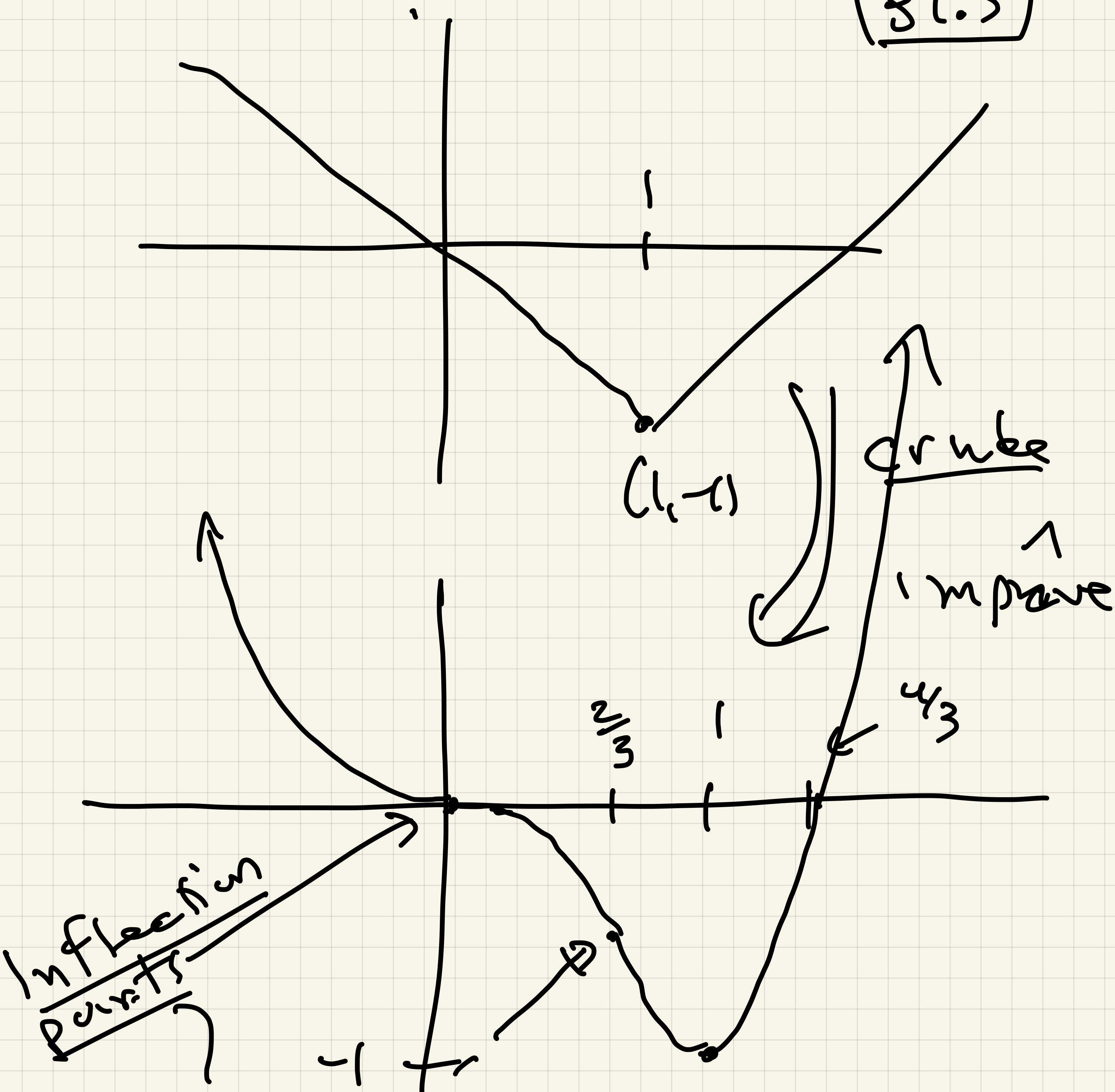
$f'' > 0$

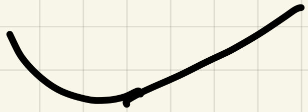
f concave up on $(-\infty, 0) \cup (2/3, \infty)$

concave down $(0, 2/3)$

<u>Make a better sketch:</u>

$\boxed{\S 4.3}$





$\left(\frac{2}{3}, -\frac{16}{27}\right)$

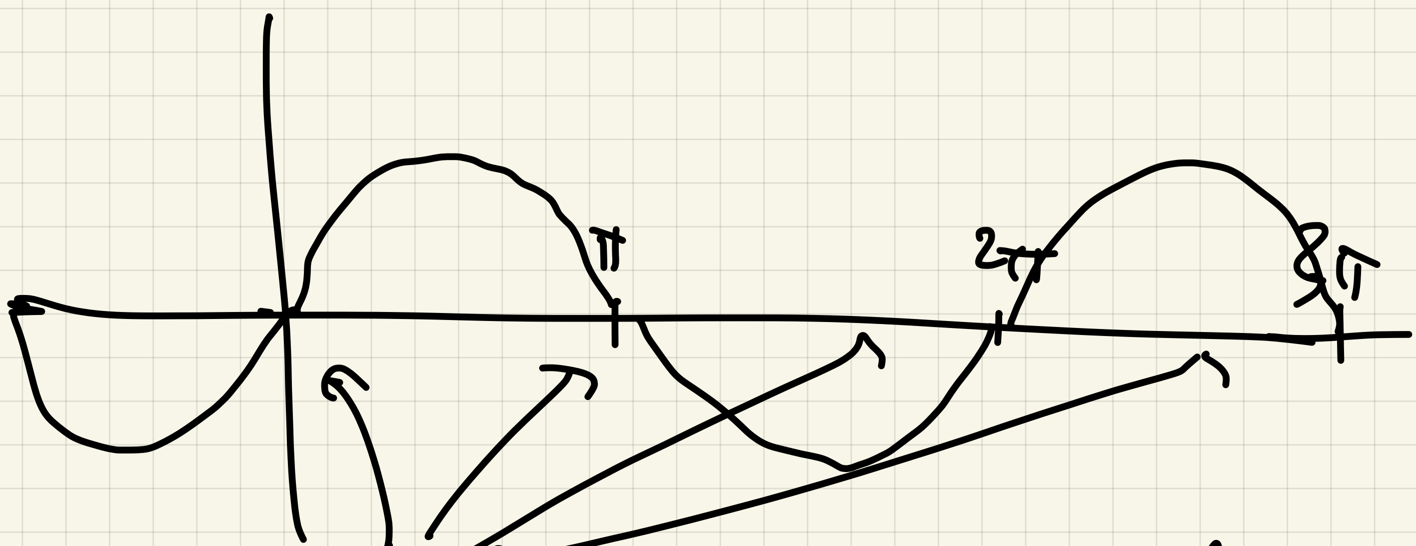
<u>Defn</u> $(c, f(c))$ is an <u>inflection point</u> if concavity changes at $(c, f(c))$ & $f'(c)$ exist

<u>Ex 8</u>

$y = \sin x$

$y' = \cos x$

$y'' = -\sin x$



Inflection points are $(n\pi, 0)$

n integer

Ex3 Find intervals where

(A) f increasing | decreasing,

(B) f concave up / down

(C) sketch

$f(x) = x^2 e^x$

(A) $f'(x) = 2xe^x + x^2 e^x$

$= \boxed{(2x + x^2)e^x}$ $\quad e^x > 0$

$2x + x^2$: $\quad x(2+x) = 0$

$x = 0, -2$

$-2 \qquad\qquad 0$

$f' > 0 \qquad f'(-1) < 0 \qquad f' > 0$

$$\begin{pmatrix} -2 & \text{rel max} \\ 0 & \text{rel min} \end{pmatrix}$$

(B) $f'' = ??$ $\qquad f' = (2x + x^2)e^x$

$$f'' = (2 + 2x)e^x + (2x + x^2)e^x$$

$$= \boxed{(x^2 + 4x + 2)e^x}$$

$= 0\ ??$

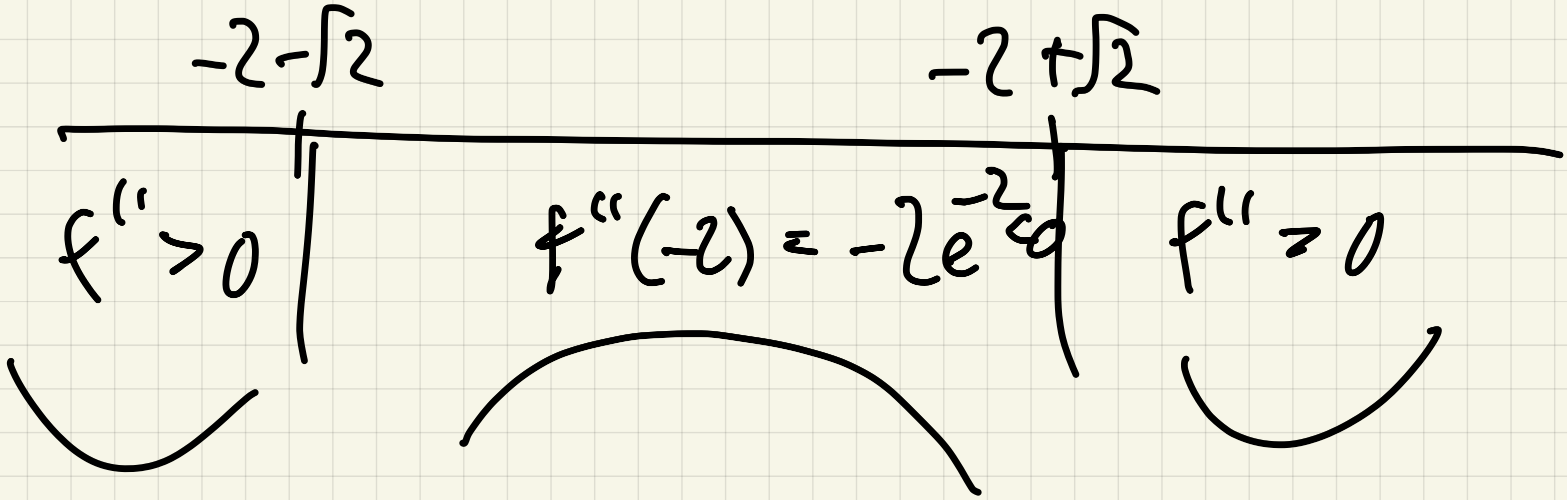
$$\underbrace{1}_{a} x^2 + \underbrace{4}_{b} x + \underbrace{2}_{c} = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(2)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{8}}{2} =$$

$$\frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

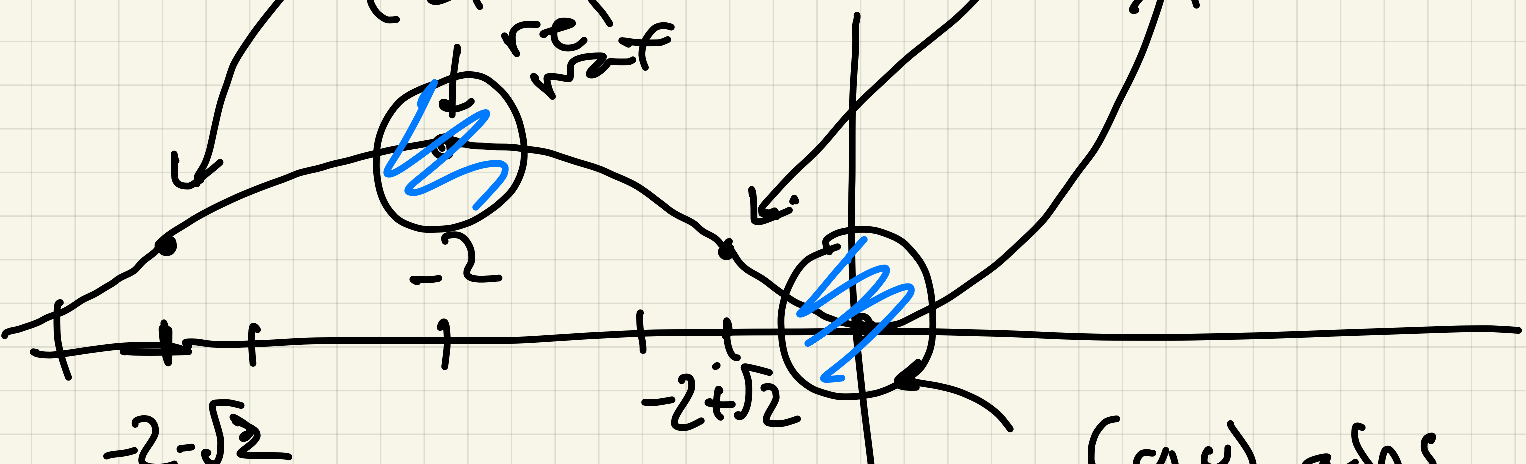


Inflection pts

$(-2-\sqrt{2}, .385)$

$(-2+\sqrt{2}, .191)$

$(-2, .541)$ rel max



$(0,0)$ abs min

Second derivative test

Suppose $f'(c) = 0$

① $f''(c) < 0 \Rightarrow (c, f(c))$ rel max

② $f''(c) > 0 \Rightarrow (c, f(c))$ rel min

③ $f''(c)=0 \rightarrow$ nothing

Ex 4 Find relative extrema for $y=x^5-80x$ using 2nd deriv test.

① **crit pts**: $y'=5x^4-80$

$= 5(x^4-16)=0$

$(x^2+4)(x^2-4)$

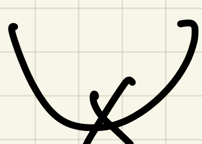
$(x-2)(x+2)$

crit pts. $\boxed{x=\pm 2}$

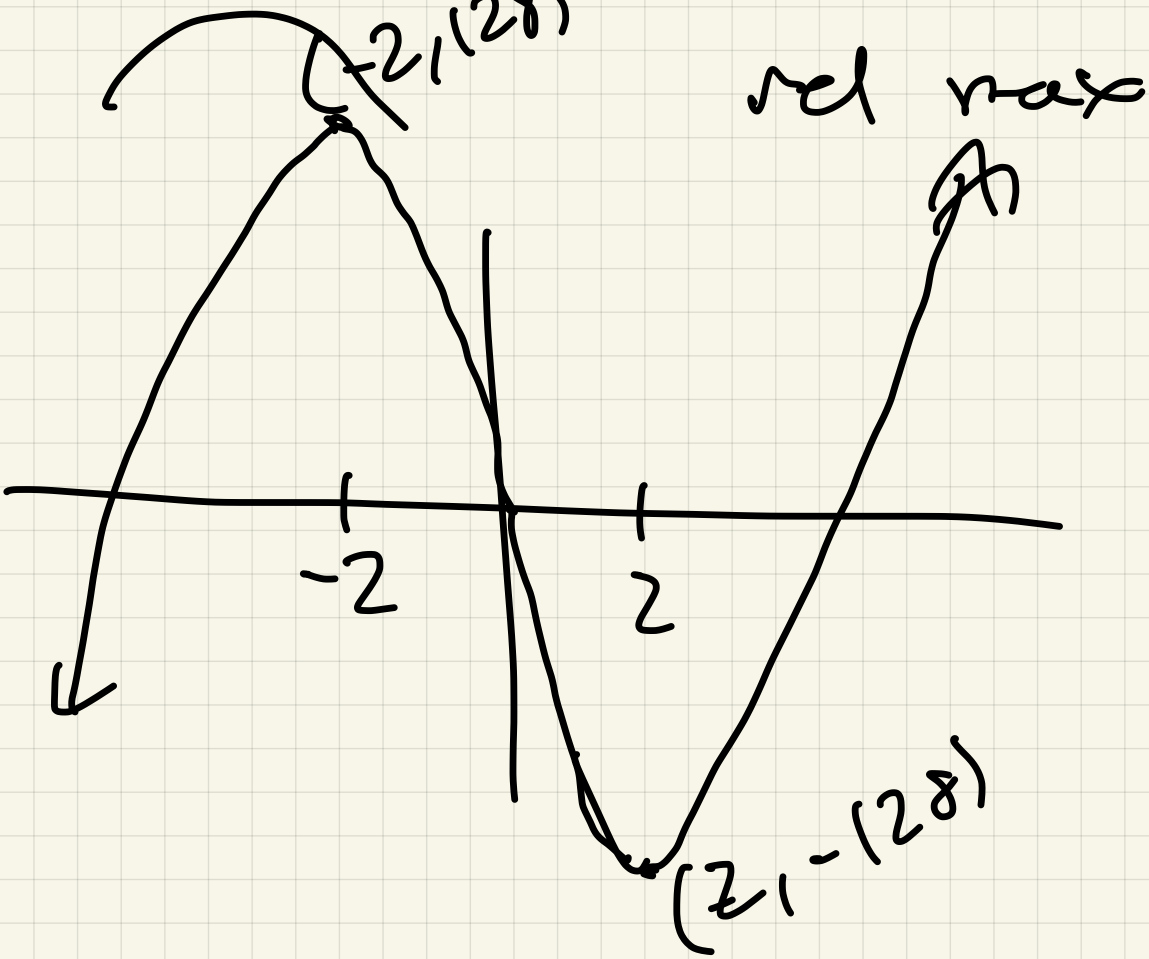
$y''=20x^3$

@ $x=2$ $\quad y''=20(2)^3=160>0$

rel min

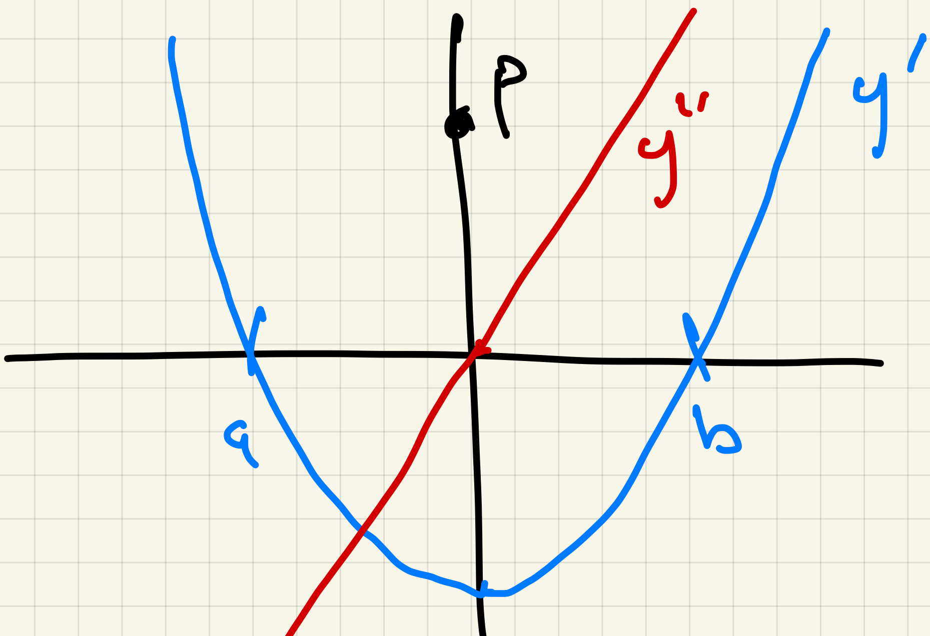


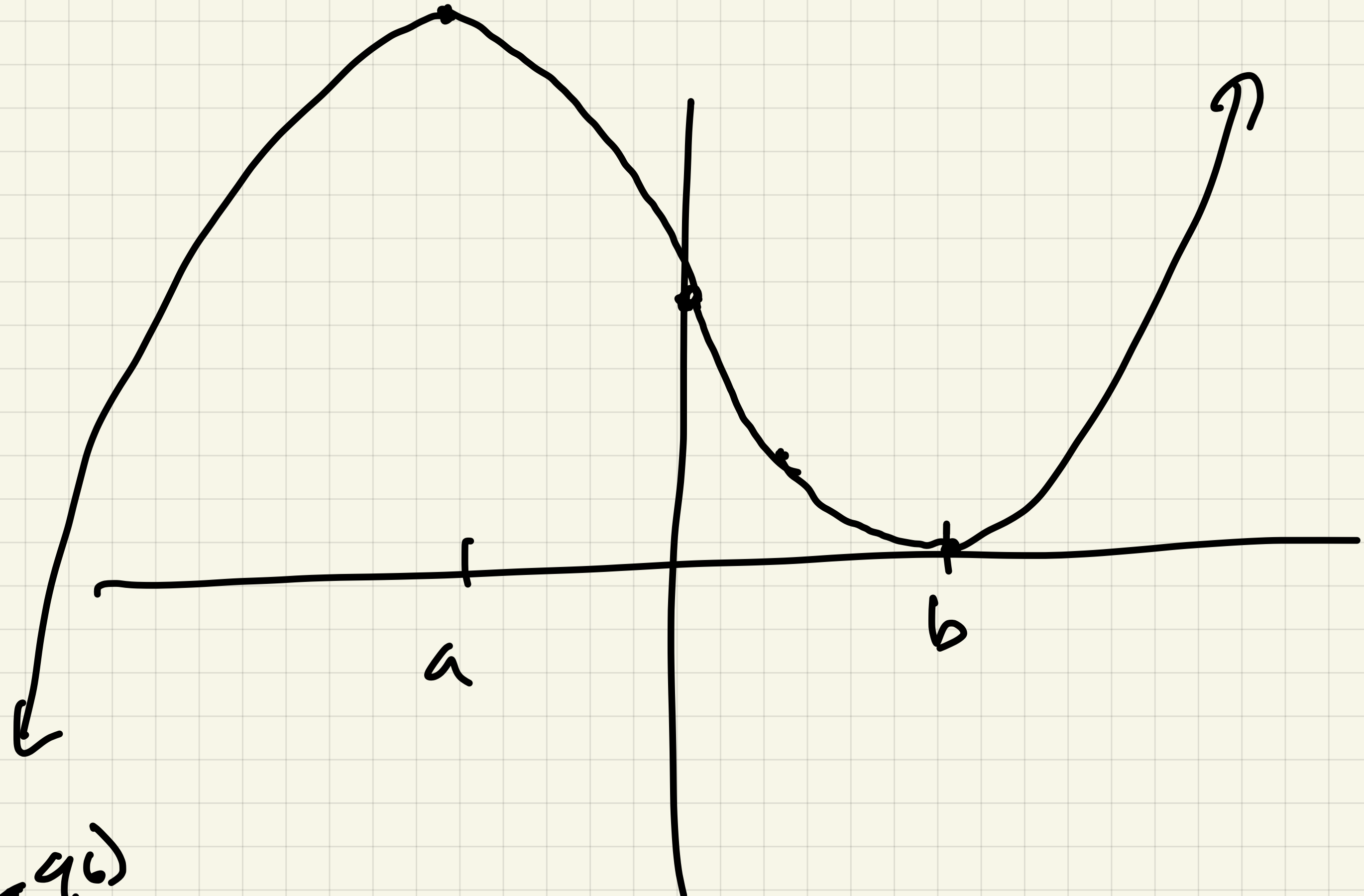
@ $x=-2$ $\quad y''=20(-2)^3=-160<0$



<u>Ex 5</u> Given graph of $y = f'$ and $y = f''$, <u>draw</u> graph of f passing through P

(#93)





(#9c)