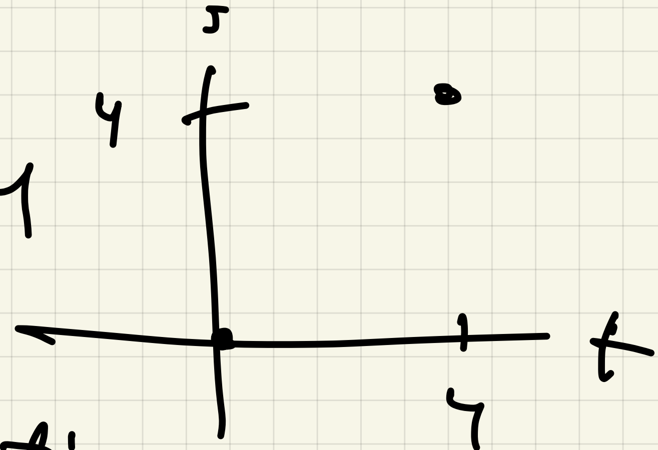
3/19/ Calc : Quiz 13

time $0 \le t \le 4$ seconds

$s(t) = t^3 - 6t^2 + 9t$ cm

1. $s(4) - s(0) =$

$4 - 0 = 4$



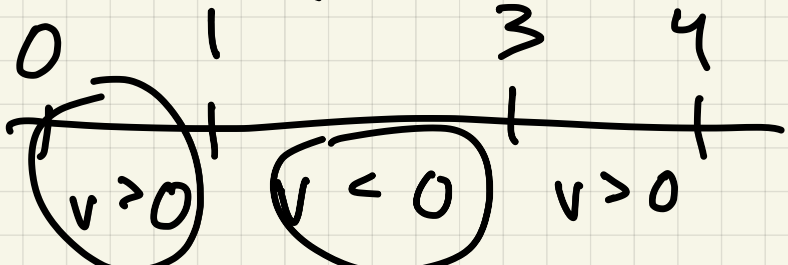
2. velocity function

$v(t) = \frac{ds}{dt} = 3t^2 - 12t + 9$

3. $v > 0$? $v < 0$

$3(t^2 - 4t + 3)$

$3(t-1)(t-3) = 0$ at $t = 1, 3$



changes direction at $t=1,3$

4. $s(1)-s(0)=4 \quad 0<t<1$

$s(1)-s(3)=4 \quad 1<t<3$

$s(4)-s(3)=4 \quad 3<t<4$

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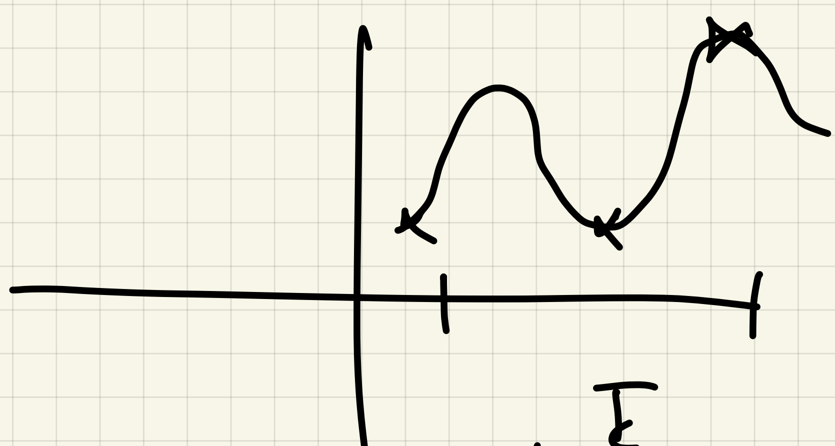
Exam 2 Thursday

Last time: $y=f(x)$ function on an interval I



absolute maximum/absolute minimum

abs max/abs min

rel max / rel min

<u>Thm 1</u> If $y = f(x)$ is <u>continuous</u> on <u>closed</u> interval $I = [a,b]$ then $f(x)$ has an abs min and abs min.

<u>Thm 2</u> If $f(x)$ has rel max/min at $x = c$, then c is a <u>critical point for $f(x)$</u>

$f'(c) = 0$

or

$f'(c)$ DNE

Thm 2 $\Rightarrow$ method for find

abs ~~max~~ mins (from Thm)

for $f(x)$ on $[a,b]$

① Evaluate $f(x)$ at each crit pt. $a<c<b$

② Evaluate $f(x)$ at a and b (endpoints)

③ Compare, find biggest/smallest

$\boxed{\text{Ex}}$ (a) Find critical points for $f(x) = x^3 - 3x^2$ ←

(b) Find abs max/min for $f(x)$ on $[-1,4]$

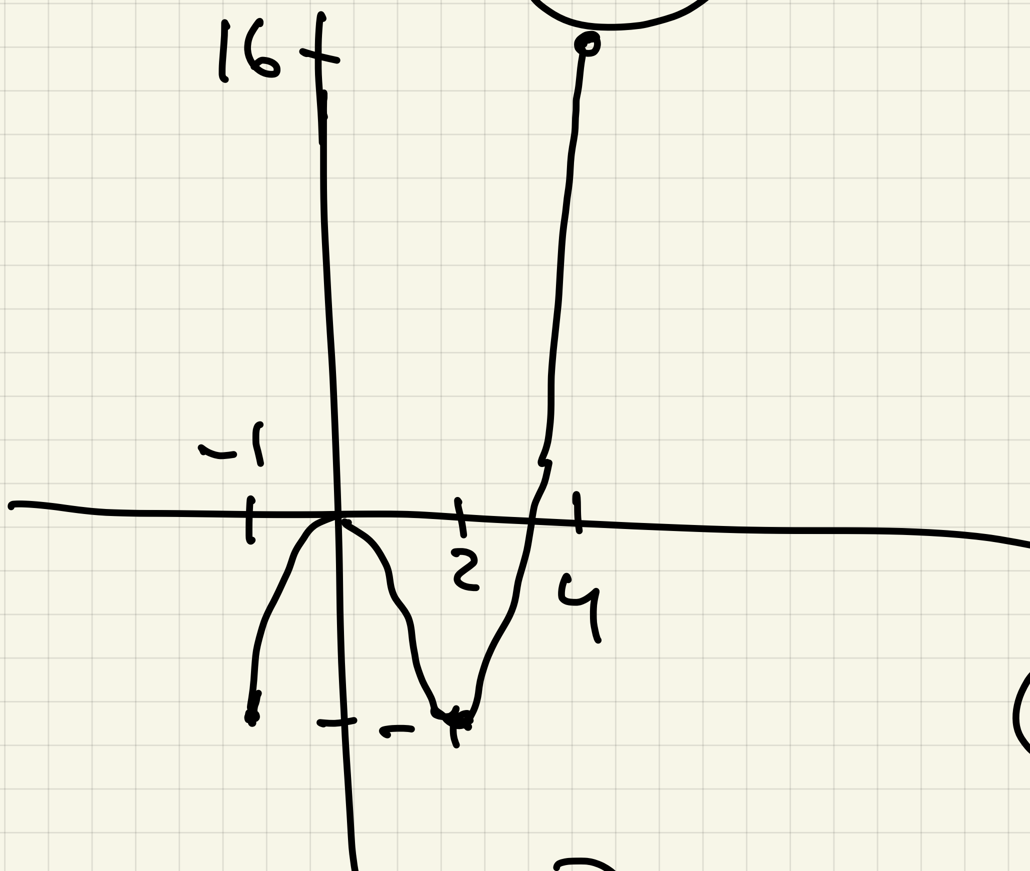
(a) $f' = 3x^2 - 6x$ exists ✓

$f'(x) = 3x(x-2) = 0 \rightarrow x = 0, 2$

$f(0) = 0$

$f(2) = -4$ abs min

endpts $\begin{cases} f(-1) = -4 \\ f(4) = 16 \end{cases}$ abs max



$(1+x^2)^2$

Ex2 $f = \dfrac{x^2}{(1+x^2)^2}$ on $[-100, 2]$

$$f'(x) = \frac{(1+x^2)^2 \cdot 2x - x^2 \cdot 2(1+x^2)^1 \cdot 2x}{(1+x^2)^4}$$

cancel $(1+x^2)$

$$= \frac{(1+x^2)\cdot 2x - 4x^3}{(1+x^2)^3}$$

$$= \frac{2x + 2x^3 - 4x^3}{(1+x^2)^3} =$$

$$\frac{2x - 2x^3}{(1+x^2)^3} = \frac{2x(1-x^2)}{(1+x^2)^3} =$$

$$\frac{2x(1-x)(1+x)}{(1+x^2)^3} = 0 \text{ at}$$

$$x = 0, 1, -1$$

Crit Pts

Evaluate

$$\frac{x^2}{(1+x^2)^2}$$

$f(0) = 0$ abs min

$f(1) = 1/4 = .25$ abs

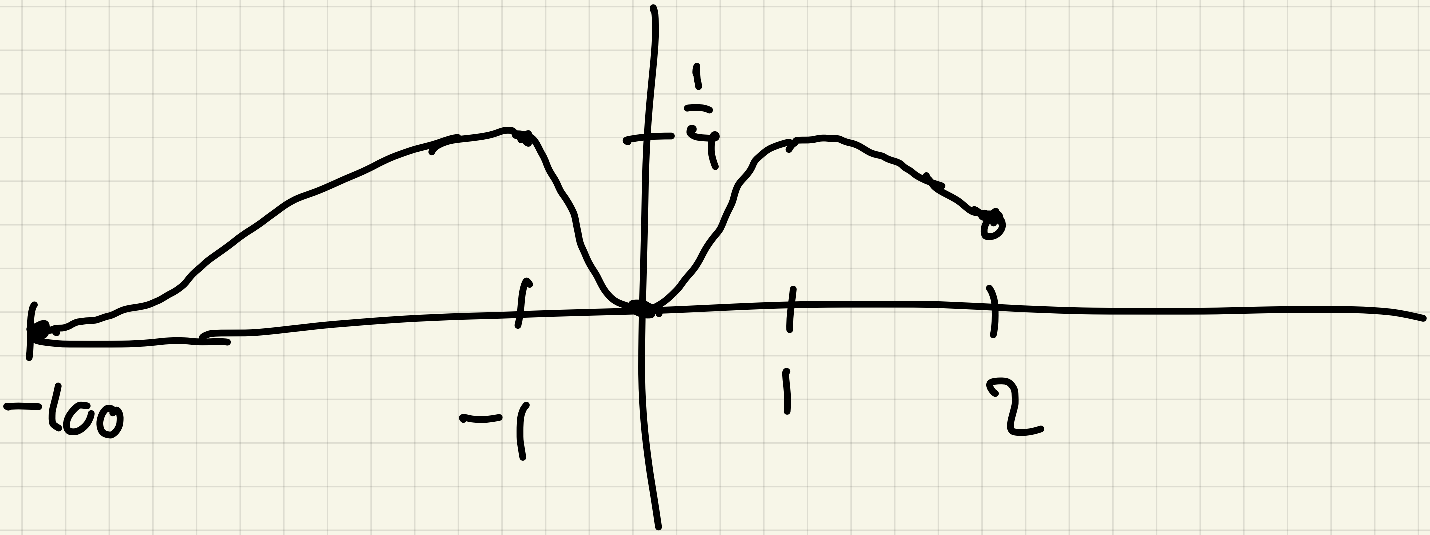
$f(-1) = 1/4 = .25$ max

$f(2) = 4/25 = .16$

$$f(-100) = \frac{10000}{(10001)^2}$$

$9.998 \times 10^{-5} = $

$.00009998...$



Ex 3 $\boxed{g = \cos^2 x + \sin x}$ on $[0, 2\pi]$

$g'(x) = 2\cos x \cdot (-\sin x) + \cos x$

$(-2\sin x + 1)\cos x = 0$

$\cos x = 0 \implies$ $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$-2\sin x + 1 = 0$

$1 = 2\sin x$

$\frac{1}{2} = \sin x$

$x = \pi/6, 5\pi/6$

Evaluation:

$$g(\pi/6) = \cos^2 \pi/6 + \sin \pi/6$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{2} = 5/4$$

$$g(5\pi/6) = \left(-\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{2} = 5/4$$

abs max

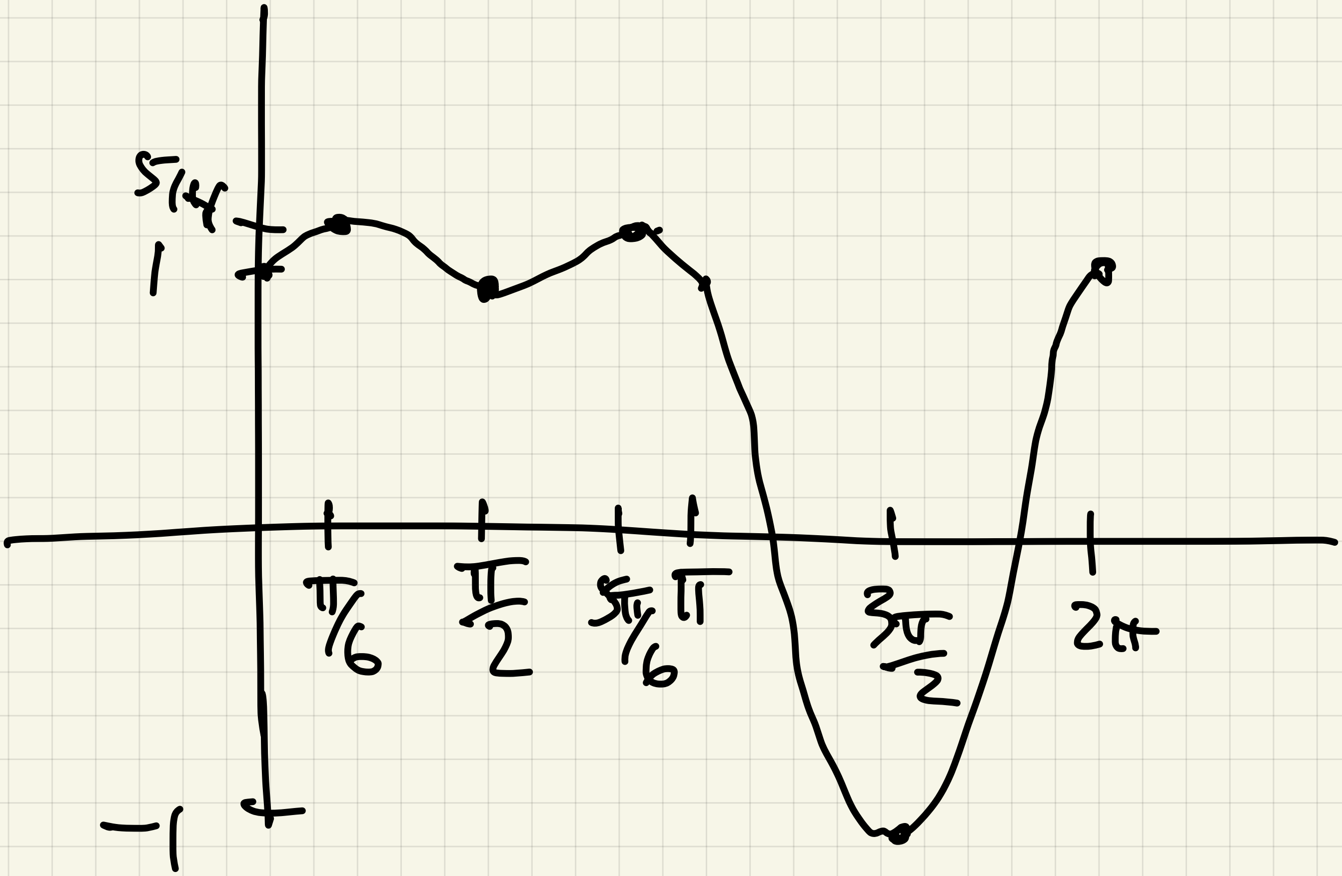
$$g\left(\frac{\pi}{2}\right) = 0^2 + 1 = 1$$

$$g\left(\frac{3\pi}{2}\right) = 0^2 + (-1) = -1$$

abs min

$$g(0) = 1^2 + 0 = 1$$

$$g(2\pi) = 1^2 + 0 = 1$$



Skip §4.2

§4.3

Defn

I interval

$y = f(x)$ function

① increases on I if for any

x_1, x_2 in I, $x_1 < x_2$,

$f(x_1) < f(x_2)$

② Decreases on I " " "

" " " "

$f(x_1) > f(x_2)$

Connection to derivatives

① If $f'(x) > 0$ on $(a,b) \Rightarrow$

f increases on (a,b)

② If $f'(x) < 0$ on $(a,b) \Rightarrow$

f decreases on (a,b)

③ If $f'(x) = 0$ on $(a,b) \Rightarrow$

f is constant

Ex) Find intervals where

$f(x)$ is increasing/decreasing

(a) $f(x) = x^3 - 3x^2$

$f' = 3x^2 - 6x = \boxed{3x(x-2)}$

$f' = 0$ at $x = 0, 2$

$0 \qquad 2$

$f'(-1) \qquad f'(1) \qquad f'(3)$

$9 > 0 \qquad -3 < 0 \qquad 9 > 0$

$+ \qquad - \qquad +$

Answer: f incr on $(-\infty, 0) \cup (2, \infty)$

f decr on $(0, 2)$

(b) $f(x) = 3x^4 - 4x^3$

$f'(x) = 12x^3 - 12x^2$

$= 12x^2(x-1) = 0$

$x = 0, 1$ crit pts

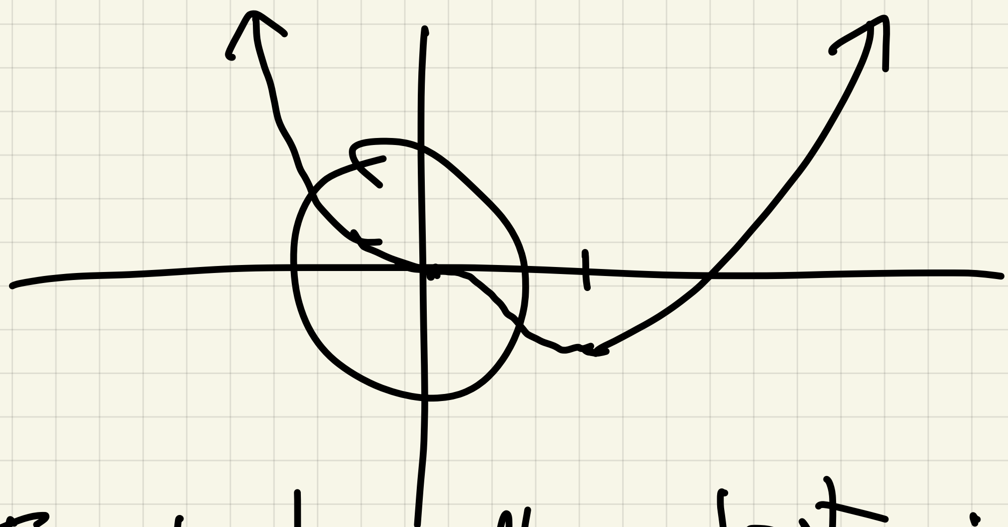
$0 \qquad 1$

$f' < 0 \qquad f' < 0 \qquad f' > 0$

$-$ $-$ $+$

f decr on $(-\infty, 1)$

f incr on $(1, \infty)$



First derivative test:

1. If $f'(x)$ changes from $-$ to $+$ at $x = c$

   $f(c)$ rel max

2. $f'(x)$ changes from $+$ to $-$ then rel min

3. f' has same sign

Then $f(c)$ neither rel ~~max~~
rel min