3/1 / Calc 1

Last time $\frac{d}{dx} \ln|x| = \frac{1}{x}$

$\rightarrow$

$\frac{d}{dx} \log_a|x| = \frac{1}{x \ln a}$

$\frac{d}{dx}(a^x) = a^x \cdot \ln a$

y	y'
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\operatorname{arcsec} x$	$\frac{1}{\lvert x\rvert \sqrt{x^2-1}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\operatorname{arccot} x$	$\frac{-1}{1+x^2}$
$\operatorname{arccsc} x$	$\frac{-1}{\lvert x\rvert \sqrt{x^2-1}}$

Ex1 (a) $y = \arcsin(\ln x)$

$$y' = \frac{1}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x}$$

$$= \frac{1}{\sqrt{1-\ln^2 x}} \cdot \frac{1}{x}$$

(b) $y = \ln^5 x = (\ln x)^5$

$$y' = 5(\ln x)^4 \cdot \frac{1}{x}$$

$$= \frac{5\ln^4 x}{x}$$

(c) $y = \ln(\arctan x)$

$$\frac{1}{\arctan x} \cdot \frac{1}{(1+x^2)}$$

(d) $y = \operatorname{arcsec}(2x+1)$

$$\frac{dy}{dx} = \frac{1}{|2x+1|\sqrt{(2x+1)^2-1}} \cdot 2$$

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§3.9/3.10

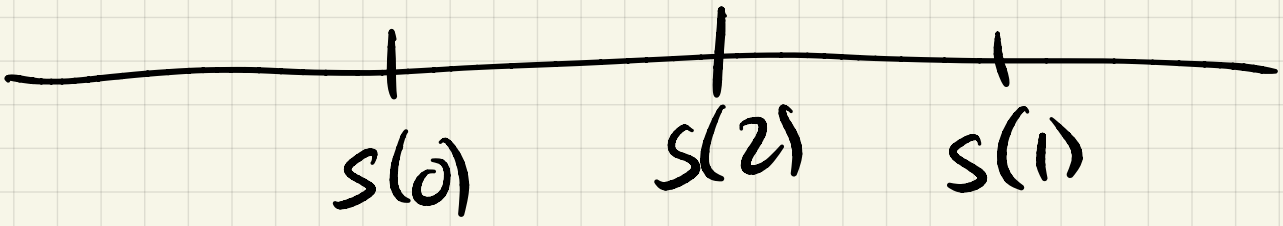
Physics Applications

§3.9 Variable t = time

1-dimensional motion:

If $s(t)$ = position of an object at Time along

x-axis



$\frac{ds}{dt}$ = velocity ; speed = $\left|\frac{ds}{dt}\right|$

$\frac{d^2s}{dt^2}$ = acceleration

$\frac{d^3s}{dt^2}$ = jerk

Displacement from time to to t_1

is $s(t_1) - s(t_0)$

Ex 1 Position of a particle moving along x-axis is

$s(t) = t^2 - 3t + 2 \quad 0 \le t \le 2$

t in seconds $= s$

s in meters $= m$

(a) Find displacement and

average velocity $=$

~~df~~ distance travelled / time elapsed

$s(2) - s(0) = 0 - 2 = -2$

average velocity $\frac{-2}{2} = -1 \text{ m/s}$

(b) Find the velocity/speed at times $t = 0$ & $t = 2$

$\frac{ds}{dt} = 2t - 3$ $\quad t=0 \rightarrow -3 \text{ m/s}$

$\quad t=2 \rightarrow 3 \text{ m/s}$

speed $= |3| = 3$ m/s

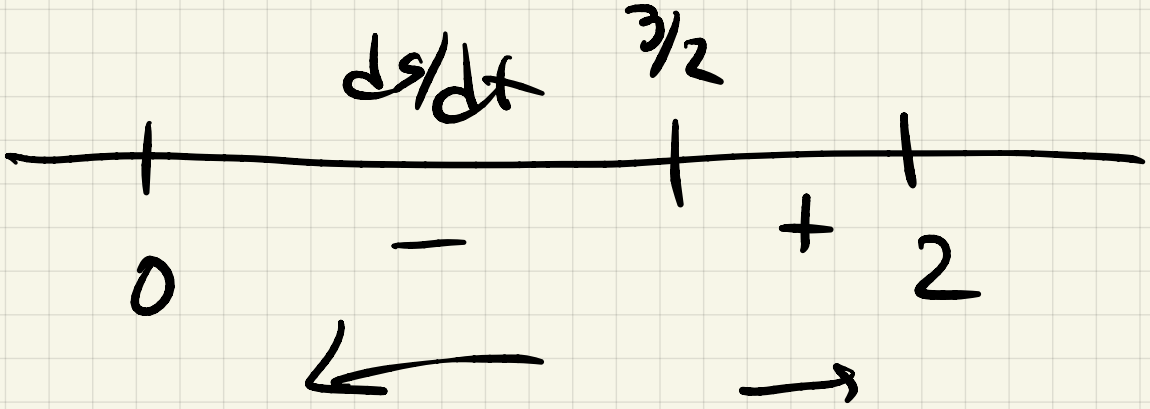
$$\frac{d^2s}{dt^2} = 2 \text{ m/s}^2$$

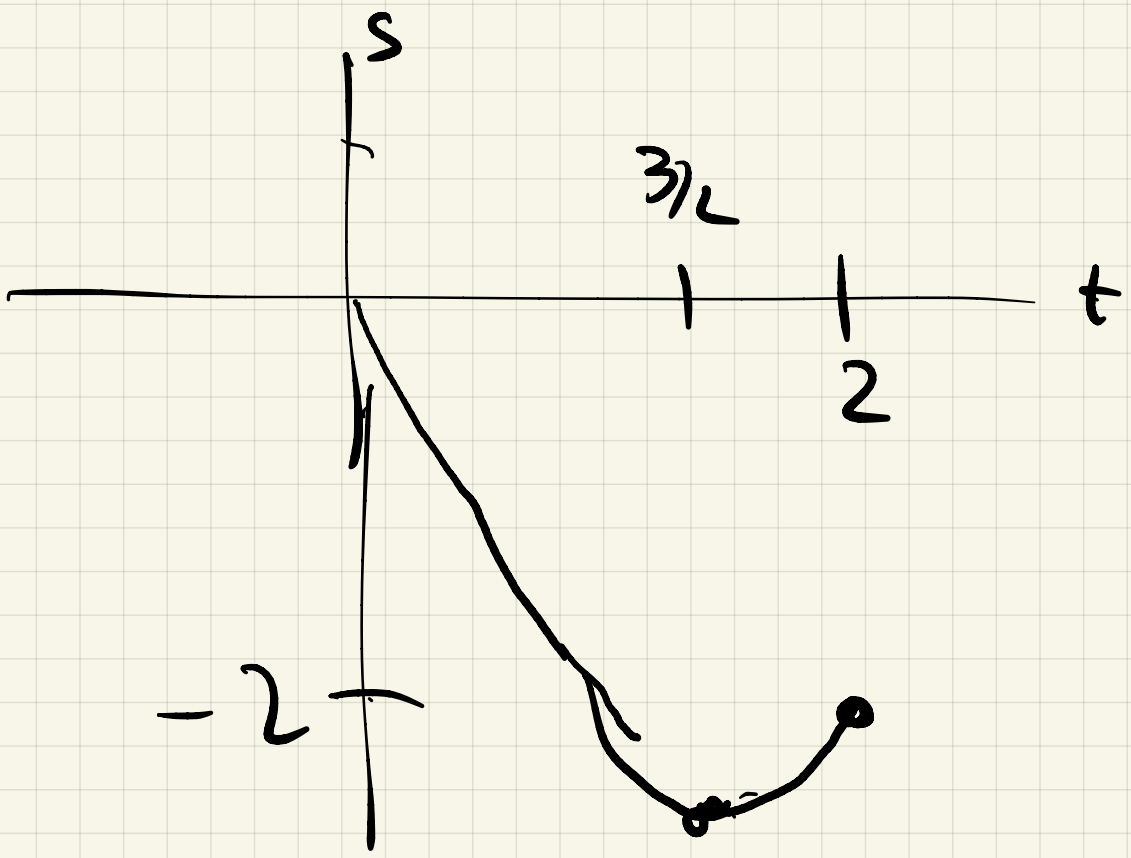
(c) When does particle change direction?

$$v = \frac{ds}{dt} = 2t - 3$$

$v > 0 \iff 2t - 3 > 0 \quad t > 3/2$

$v < 0 \iff \quad t < 3/2$





Ex 2: A rock is catapulted into the air at time $t=0$, and its height is given by $h(t) = 96t - 16t^2$ ft

$t \geq 0$

seconds

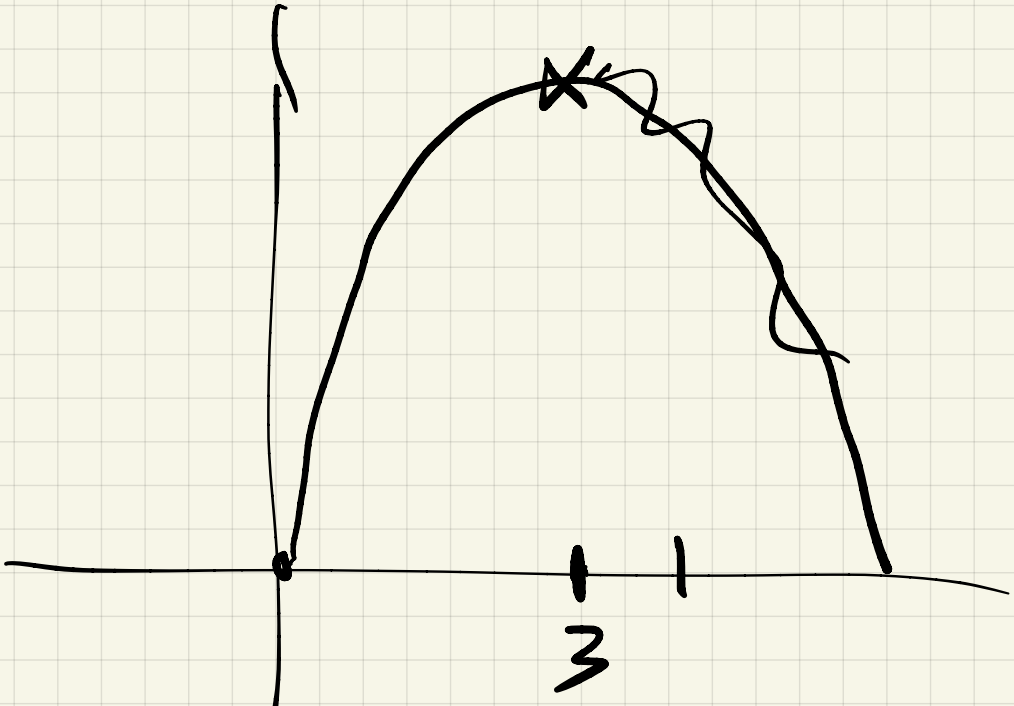
Find:

(a) Initial velocity

(b) velocity at $t = 4$

(c) When does rock begin to return to ground

(d) When does it land?



(a) $\frac{dh}{dt} = 96 - 32t$

$\frac{dh}{dt}(0) = 96$ ft/sec

(b) $\frac{dh}{dt}(4) = 96 - 32(4) =$

$96 - 128 = -32$ ft/sec

(c) $96 - 32t > 0$

$96 > 32t$

$\boxed{3 > t}$ $v > 0$

$t > 3$ $v < 0$

(d) $\underline{h = 0}$:

$\underline{96t - 16t^2 = 0}$

$t(96 - 16t) = 0$

$\Downarrow$

$\boxed{t = 0}$ $96 - 16t = 0$

$t = \frac{96}{16} = 6$ sec

Maximum Height :

$$h(3) = 96(3) - 16(3)^2$$

$$288 - 144 = 144 \text{ ft}$$