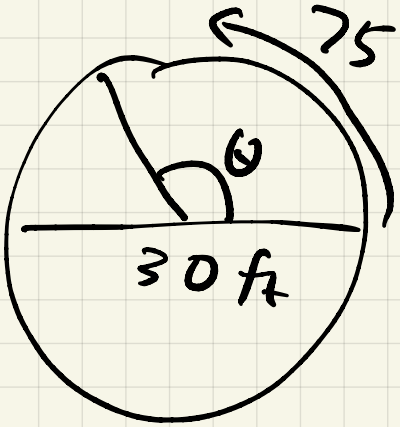
1/30/ Calc I  Quiz 3

1. 


$s = 15$

$\frac{75}{15} = 5$ radians

$5\left(\frac{180}{\pi}\right) = \left(\frac{900}{\pi}\right)^{\circ}$

2. $25^{-2/3} \cdot 5^{1/3}$

$=$

$25 \cdot 5^{-2/3 + 1/3}$

$a^x a^y = a^{x+y}$

$(ab)^x = a^x b^y$

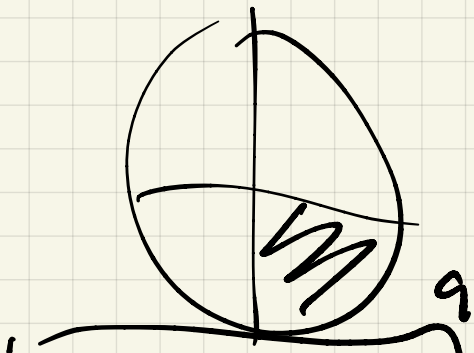
$(a^x)^y = a^{xy}$

$(5^2)^{-2/3} 5^{1/3} =$

$5^{-4/3} \cdot 5^{1/3} = 5^{-4/3+1/3} = 5^{-1}$

$= \frac{1}{5}$

3. $\cos\theta = 3/5 \quad 3\pi/2 \le \theta \le 2\pi$



quad 4

$\sin\theta < 0$

$\tan\theta < 0$

$\sin^2\theta + \cos^2\theta = 1$

$(\sin\theta)^2 + (3/5)^2 = 1$

$+ 9/25 = 25/25$

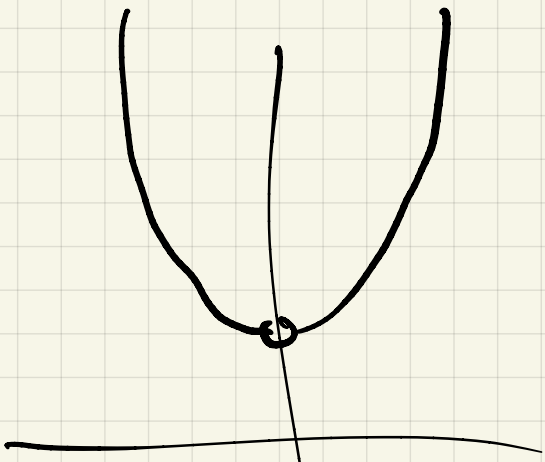
$(\sin\theta)^2 = 16/25$

$\sin\theta = \pm 4/5$

$\sin\theta = -4/5$

$\tan\theta = -4/3$

## §2.2



$y = \frac{x}{\sin x}$

$x \to 0$

$y \to ??$

**Defn:** $y = f(x)$

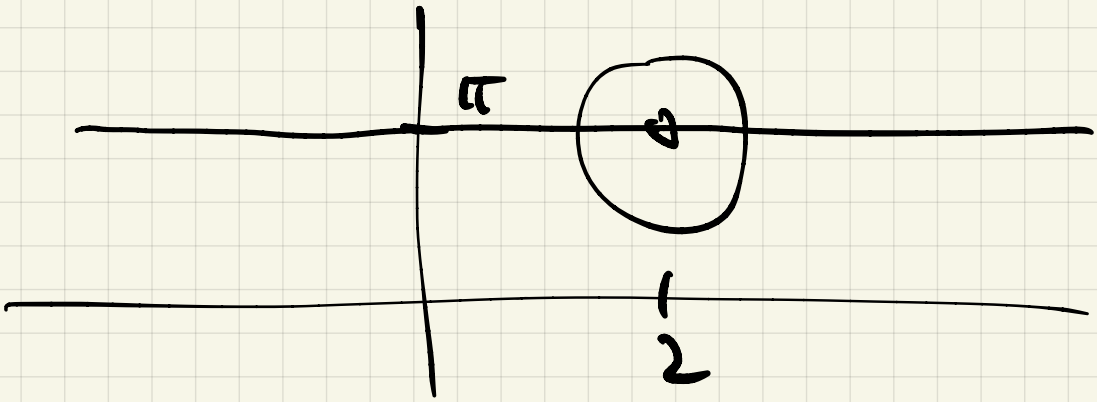
$f$ approaches the limit $L$ as $x$ approaches $c$ if we can make $f(x)$ as close to $L$ as we like by choosing $x$ close enough

to $c$ (but $x \neq c$)

**Notation:** $\lim_{x \to c} f(x) = L$

**Notes:** value $f(c)$ irrelevant

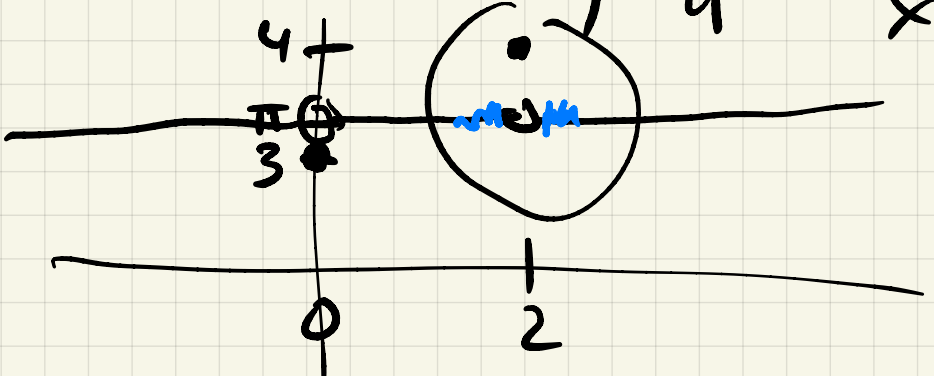
**Ex)** $f(x) = \pi$ all $x$




(a) $\lim_{x \to 2} f(x) = \pi$

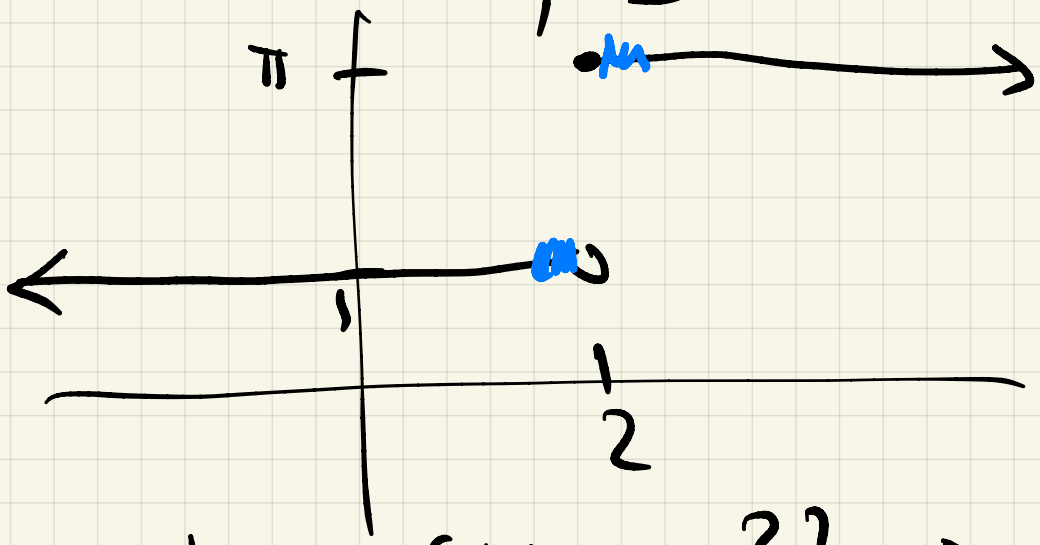
(b) $f(x) = \pi$ all $x$

(b) $f(x) = \begin{cases} \pi & x \neq 0, 2 \\ 3 & x = 0 \\ 4 & x = 2 \end{cases}$



$$\lim_{x \to 2} f(x) = \pi$$

(c) $$f(x) = \begin{cases} \pi & x \geq 2 \\ 1 & x < 2 \end{cases}$$




$$\lim_{x \to 2} f(x) = ??\ \text{DNE}$$

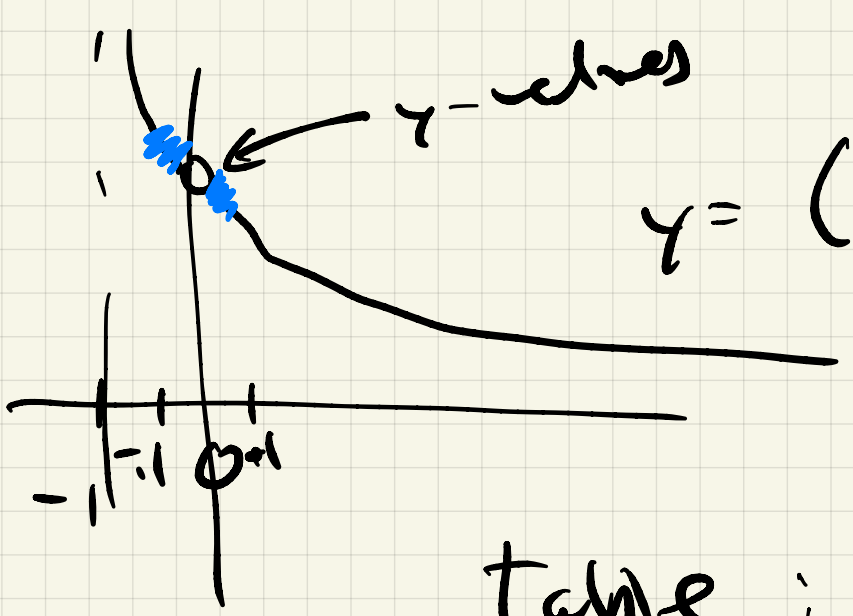
b/c $x < 2$, $f(x) = 1$

$x > 2$ $f(x) = \pi$

## Estimating limits

Ex 2 Estimate $\lim_{x \to 0} (1+x)^{3/x}$

with a calculator to 3 decimal places.




$y = (1+x)^{3/x}$ $(1.00001)^{300000}$ $(1.001)^{3000}$

table:

left

| x | f(x) |
|---|---|
| -.1 | 23.584 |
| -.001 | 20.1157 |
| -.00001 | 20.0858 |

right

| x | f(x) |
|---|---|
| .1 | 17.4494 |
| .001 | 20.0555 |
| .00001 | 20.0852 |

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get 20.085

Ex3 $\lim_{x \to 0} \sin(\pi/x)$ to 3 decimals

table

left

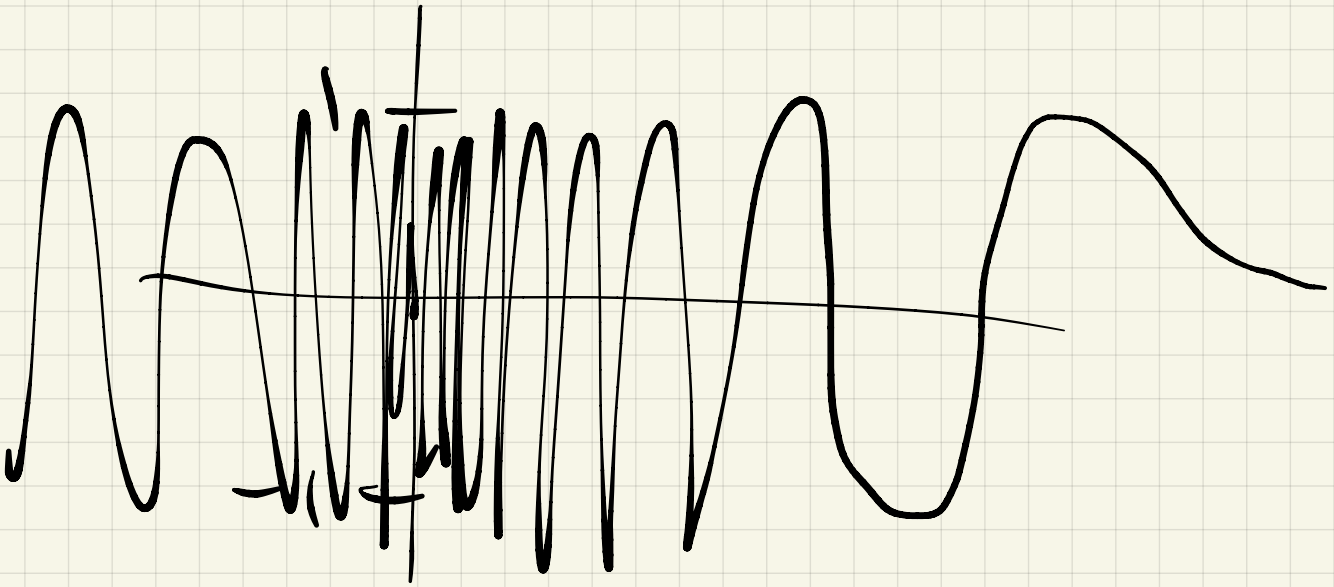
| x | f(x) |
|---|---|
| -1 | 0 |
| -.1 | 0 |

| x | |
|---|---|
| 1 | 0 |
| .1 | 0 |

| | |
|---|---|
| -.01 | 0 |
| -.001 | 0 |

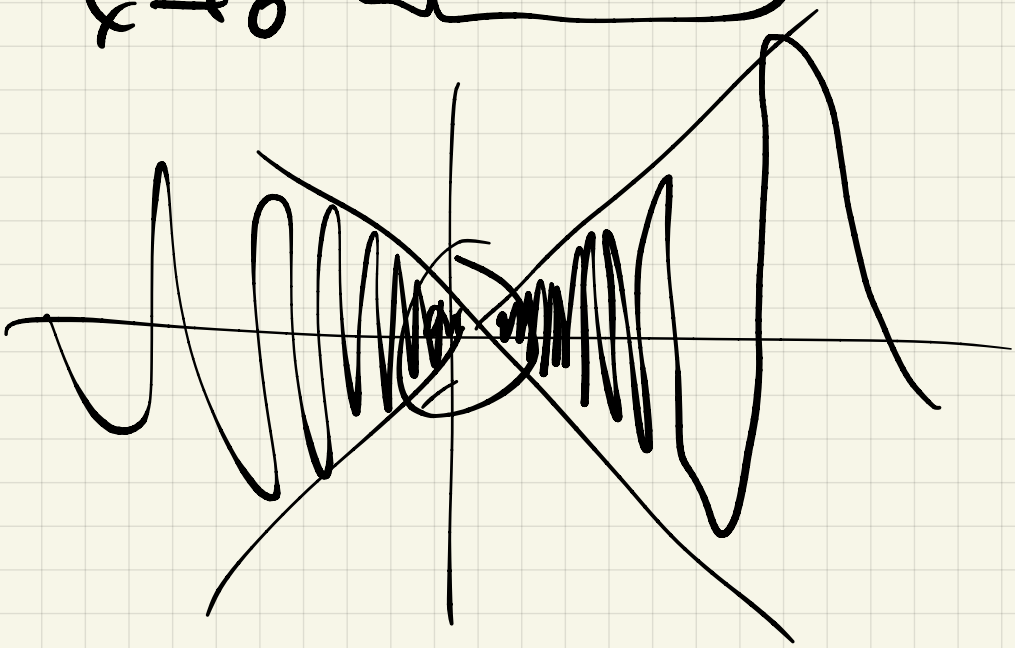
| | |
|---|---|
| .01 | 0 |
| .001 | 0 |

**Sketch** $y = \sin\left(\frac{\pi}{x}\right)$



$$\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right) \quad \text{DNE}$$

**Ex** $\lim_{x \to 0} \boxed{x \sin\left(\frac{\pi}{x}\right)}$



# Algebraic limits

## Basic limits:

$b$ constant

① $\lim_{x \to c} b = b$

② $\lim_{x \to c} x = c$

$\lim_{x \to 2} \pi = \pi$ $\quad$ $\lim_{x \to 2} x = 2$

## Combining limits

Suppose $\lim_{x \to c} f(x) = L$

$\lim_{x \to c} g(x) = K$, Then

① $\lim_{x \to c} f(x) + g(x) = L + K$

② $\lim_{x \to c} f(x) - g(x) = L - K$

③ $\lim_{x \to c} f(x) \cdot g(x) = L \cdot K$

④ $\lim_{x \to c} f(x)/g(x) = L/K$, (if $K \neq 0$)

⑤ $\lim_{x\to c} b\,f(x) = b\cdot L$ $\quad b$ const

⑥ $\lim_{x\to c} f(x)^n = L^n$ $\quad n>0$ int

⑦ $\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{L}$

- $n$ odd
- $n$ even, $L>0$

Ex 5 (a) $\lim_{x\to 2} 3x^2 - 4x$

// ②

$\lim_{x\to 2} 3x^2 - \lim_{x\to 2} 4x$

// ⑤ $\qquad$ 1. ⑤

$3\lim_{x\to 2} x^2 - 4\lim_{x\to 2} x$

// ⑥

$3\left(\lim_{x\to 2} x\right)\left(\lim_{x\to 2} x\right) - 4\cdot 2$

$3\cdot 2\cdot 2 - 4\cdot 2 =$

$12 - 8 = 4$.

(b) $\lim_{x \to 2} \dfrac{3x^2 - 4x}{x+6} =$

$$\frac{\lim_{x \to 2} 3x^2 - 4x}{\lim_{x \to 2} x+6} = \frac{4}{8} = \frac{1}{2}$$

$$\lim_{x \to 2} x + \lim_{x \to 2} 6$$

$$2 + 6 = 8$$

<u>Thm</u> If $p(x)$ and $q(x)$ are polynomials, then

① $\lim_{x \to c} p(x) = p(c)$

② $\lim_{x \to c} \dfrac{p(x)}{q(x)} = \dfrac{p(c)}{q(c)}$ $q(c) \neq 0$

Ex6 $\lim_{x \to -1} \frac{3x^4+2x+1}{4x+7} = \frac{2}{3}$

Ex7

(a) $\lim_{x \to 3} \sqrt[3]{\frac{x^2+9}{51+x}}$

$\lim_{x \to 3} \frac{x^2+9}{51+x} = \frac{18}{54} = \frac{1}{3}$

(2) $\Rightarrow \lim_{x \to 3} \sqrt[3]{\frac{x^2+9}{51+x}} = \sqrt[3]{\frac{1}{3}}$

(b) $\lim_{x \to 5} (x^2)^{-2/3} \cdot x^{1/3} =$

$(5^2)^{-2/3} \cdot 5^{1/3} = \frac{1}{5}$

(c) $\lim_{x \to c} |x| = \lim_{x \to c} \sqrt{x^2} = \sqrt{c^2} = |c|$

## Less obvious limits

(a) $\lim_{x \to 3} \frac{x-3}{3x^2-27} = ? \quad \frac{0}{0}$ undef

$$\lim_{x\to 3} \frac{(x-3)}{3(x^2-9)} = \lim_{x\to 3} \frac{(x-3)}{3(x-3)(x+3)}$$

$$\lim_{x\to 3} \frac{1}{3(x+3)}$$

$$\frac{1}{3(6)} = \frac{1}{18}$$

b/c

$$\frac{1}{3(x+3)} = \frac{x-3}{3x^2-27}$$

for x case 3

$x \neq 3$

(b) $$\lim_{x\to 1} \frac{x^3-12x+11}{(x-1)}$$ $\frac{0}{0}$

$$= \lim_{x\to 1} \frac{(x-1)(x^2+x-11)}{(x-1)} = \lim_{x\to 1} \frac{x^2+x-11}{1}$$

$$= -9$$

$$x-1 \overline{)\, x^3 - 12x + 1}$$

$$x^3 - x^2$$

$$x^2 - 12x + 1$$