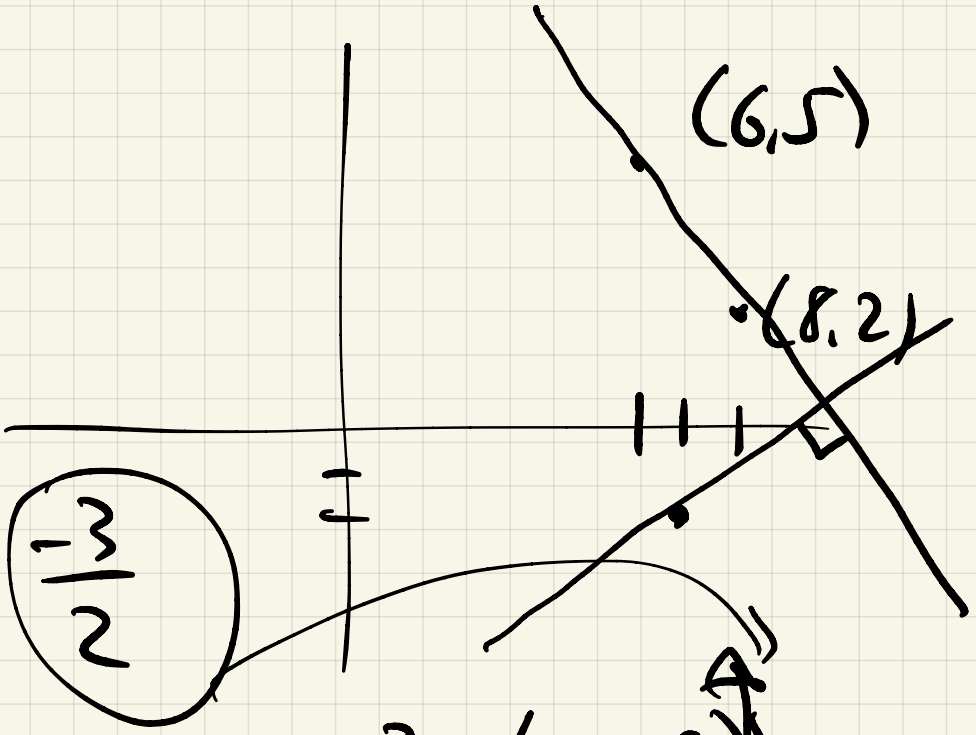
1/23/ Calc

Quiz

1.



$$m = \frac{2-5}{8-6} = \frac{-3}{2}$$

pt-slope: $y - 2 = -\frac{3}{2}(x-8)$

$$y = -\frac{3}{2}x + 14$$

2. y-int $x = 0 \Rightarrow y = 14$

x-int $y = 0 \rightarrow 0 = -\frac{3}{2}x + 14$

$$\frac{3}{2}x = 14$$

$$x = \frac{2}{3}14 = \frac{28}{3}$$

21
9.33

3. $(7, -2)$

$m = \frac{2}{3}$

⇓

$$y+2 = \tfrac{2}{3}(x-7)$$

$$y = \tfrac{2}{3}x - \tfrac{20}{3}$$

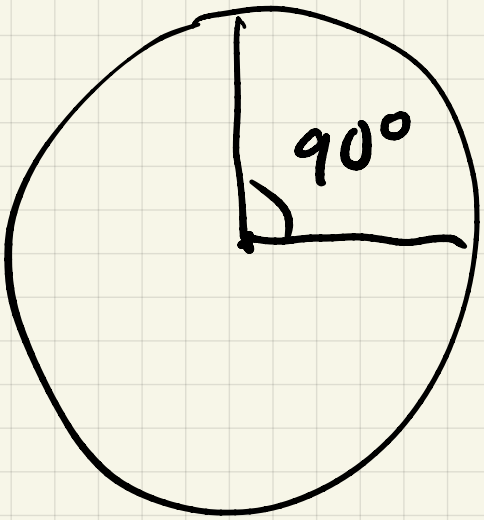
4.

Skip §1.2

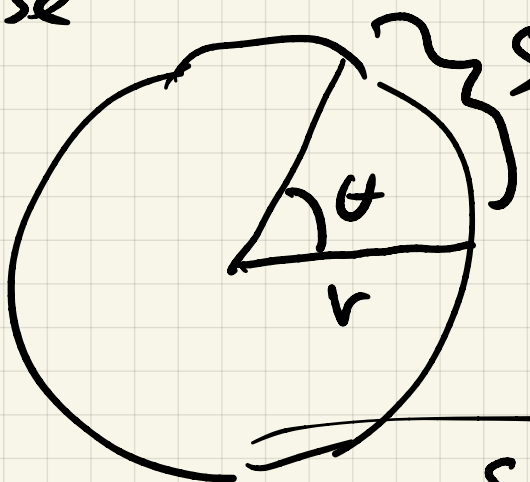
Skip §1.4 → later (discuss Friday)

§1.3 Trig functions & radian measure

Radian measure



Use



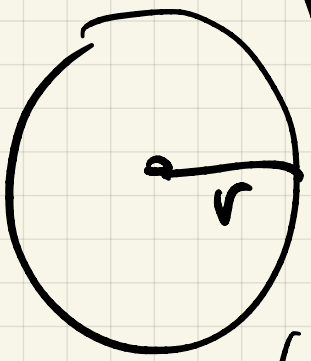
radian

$$\theta = \frac{s}{r}$$

$$s = \theta r$$

Conversions:

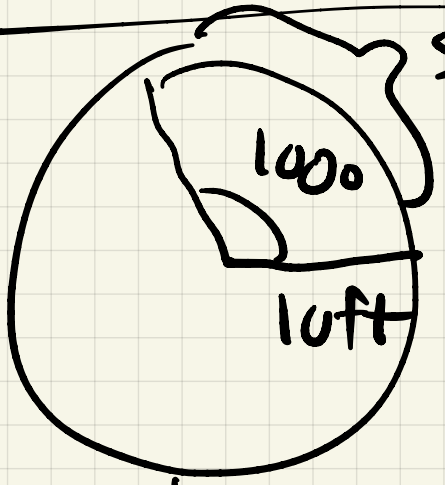
$360^\circ =$ all way around circle $= \frac{2\pi r}{r} = 2\pi^{rad}$



$$1^\circ = \frac{2\pi}{360} = \frac{\pi}{180} \text{ rads}$$

$$1 \text{ rad} = \frac{360}{2\pi} = \frac{180}{\pi}$$

Ex 1 (a)



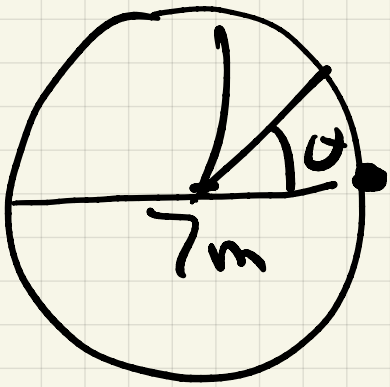
Circle radius of

Find length of arc s ?

$$100^\circ = 100\left(\frac{\pi}{180} \text{ rads}\right) = \frac{10\pi}{18} = \frac{5\pi}{9}$$

$$s = 10\text{ft} \cdot \frac{5\pi}{9} = \frac{50\pi}{9} \left(\approx 17.453\right)$$

(b)



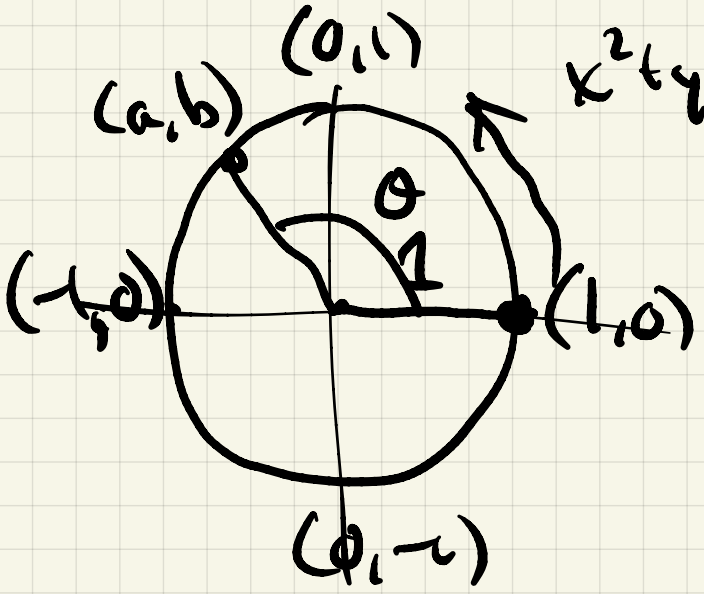
circle
diam = 7m
walk 30 m
around the
circle.

What is change in angle θ

$$\theta = \frac{s}{r} = \frac{30\,m}{3.5\,m} = \boxed{\frac{60}{7}\ \text{rads}}$$

$$\frac{60}{7}\left(\frac{180}{\pi}\right)^\circ = \frac{10800}{7\pi} \approx 491.106^\circ$$

Trig functions:



For θ in $\mathbb{R}$,
more θ units
counterclockwise
around unit

circle starting at (1,0), stop at $P = (a, b)$

Then define

$\cos\theta = a$

$\sin\theta = b$

$\tan\theta = \frac{b}{a}$

calculator

$\sec\theta = \frac{1}{a}$

$\csc\theta = \frac{1}{b}$

$\cot\theta = \frac{a}{b}$

not on calculator

Note: Calculator in radian measure

Ex 1 $\theta = 0$ $(a, b) = (1, 0)$

(a) $\cos 0 = 1$

$\sin 0 = 0$

$\tan 0 = 0$

$\sec 0 = 1$

$\csc 0 =$ undef

$\cot 0 =$ undef

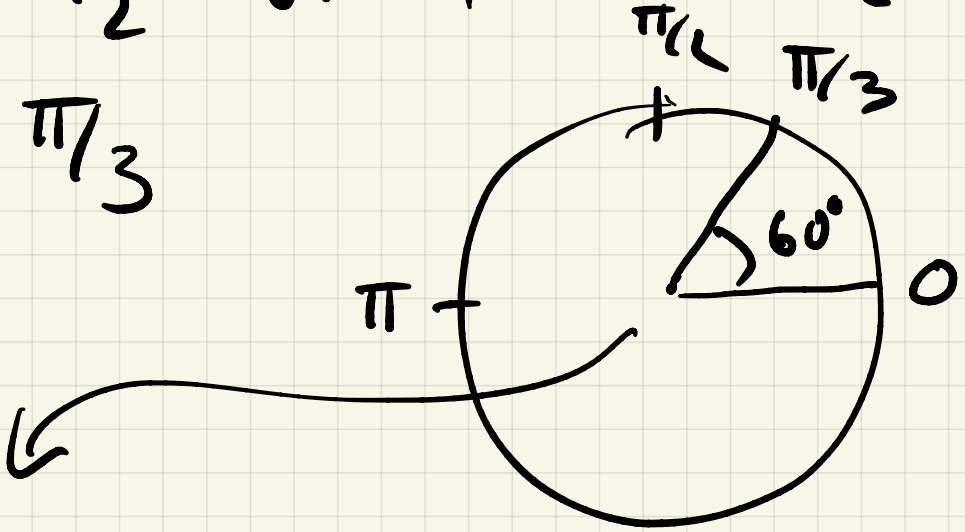
(b) $x = \frac{\pi}{2} \Rightarrow P = (0, 1)$

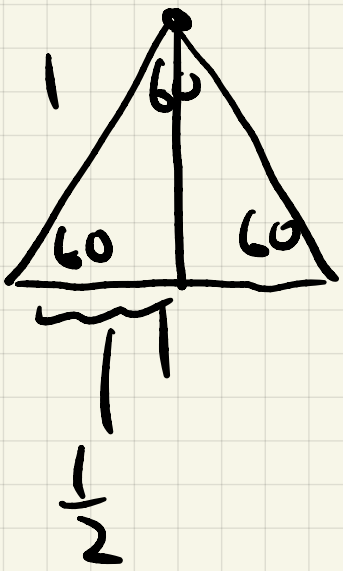
$\cos \pi/2 = 0$ $\quad \sec \pi/2 = \text{undef}$

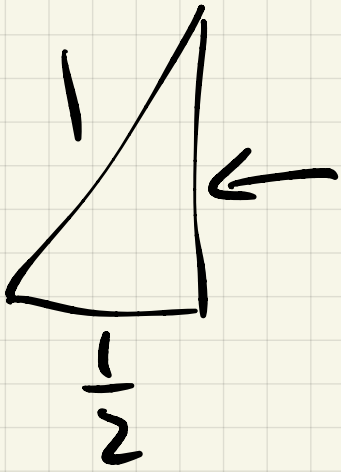
$\sin \pi/2 = 1$ $\quad \csc \pi/2 = 1$

$\tan \pi/2 = \text{undef}$ $\quad \cot \pi/2 = 0$

(c) $x = \pi/3$







$(a, b) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(\frac{1}{2}\right)^2 + b^2 = 1$

$b^2 = 1 - \frac{1}{4} = 3/4$

$b = \sqrt{3}/2$

$\cos \pi/3 = \frac{1}{2}$ $\quad \sec = 2$

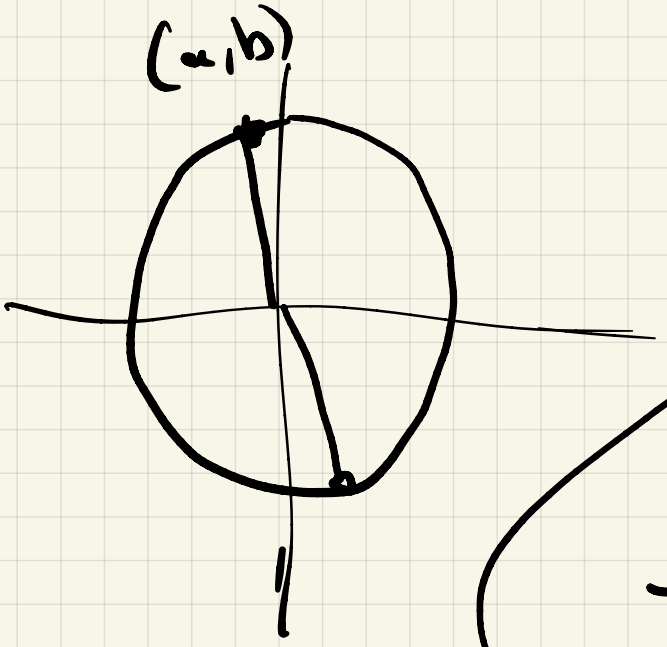
$\sin \pi/3 = \sqrt{3}/2$ $\quad \csc = 2/\sqrt{3}$

$\tan \pi/3 = \frac{1}{\sqrt{3}}$ $\quad \cot = \sqrt{3}$

Ex 2 If $\pi/2 \le \theta \le \pi$

and $\tan\theta = -2v$

find the other 5 trig functions.



$\tan\theta = -2v = \frac{b}{a}$

$a^2 + b^2 = 1$

Solve for a+b

$-2v = \frac{b}{a} \Rightarrow$

$\boxed{b = -2va}$

$a^2 + b^2 = a^2 + (-2va)^2 = 1$

$a^2 + 400a^2 = 1$

$401a^2 = 1$

$a = \frac{-1}{\sqrt{401}}$; $\cos\theta$; $a^2 = \frac{1}{401}$

$a = \pm \frac{1}{\sqrt{401}}$

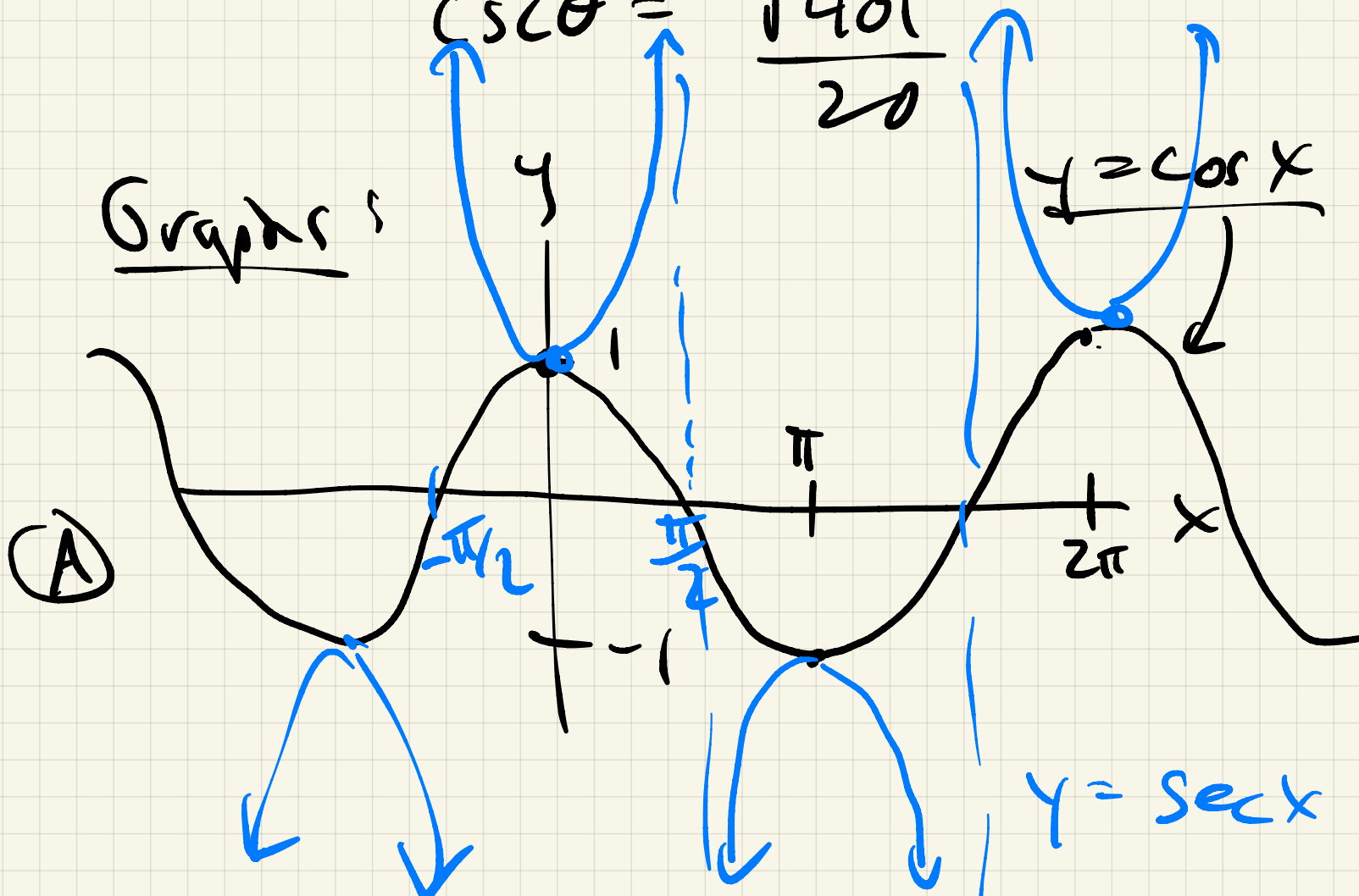
$b = \frac{20}{\sqrt{401}} = \sin\theta$

$\tan = -20$ $\quad \cot \frac{-1}{20}$

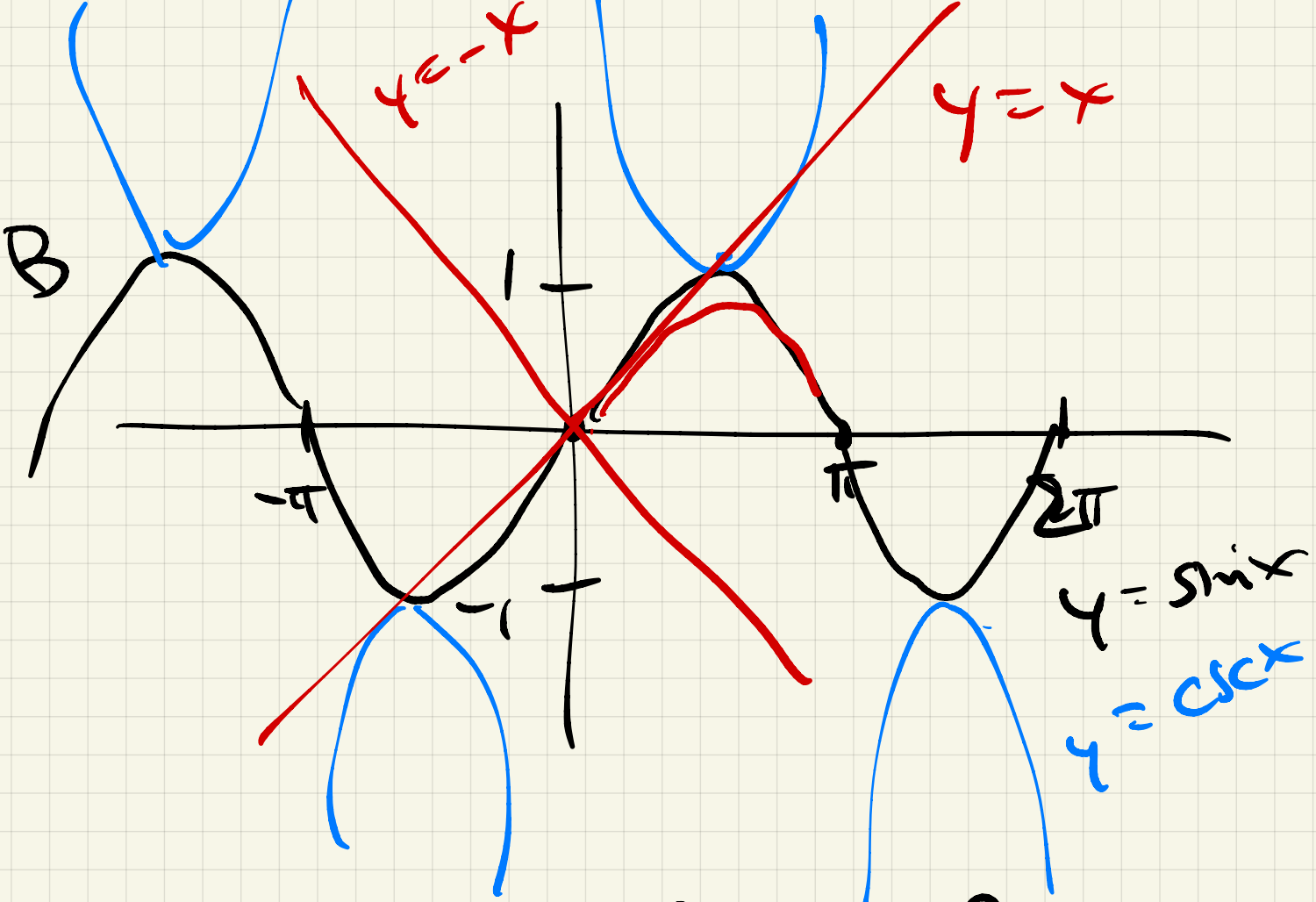
$\sec\theta = -\sqrt{401}$

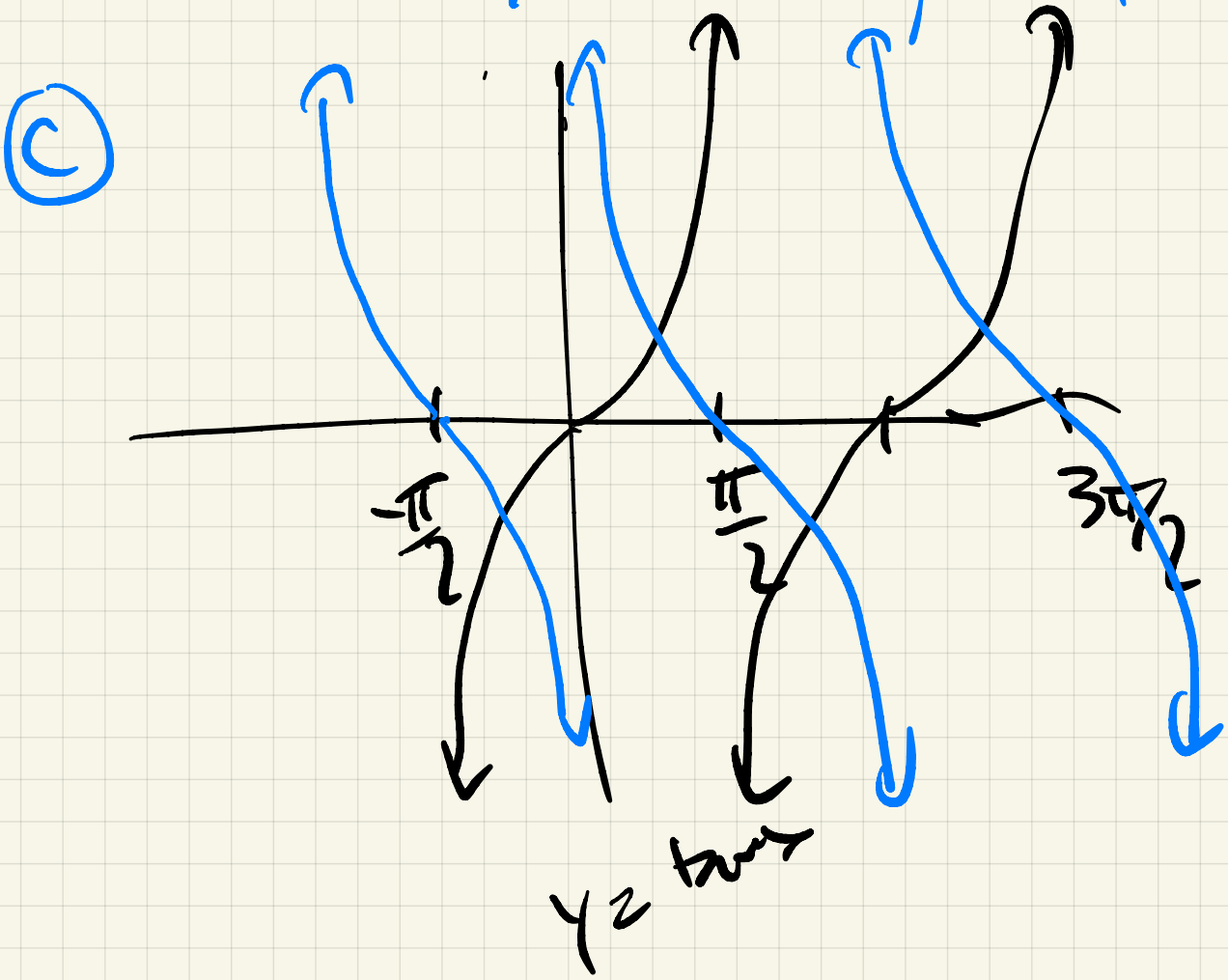
$\csc\theta = \frac{\sqrt{401}}{20}$

Graphs:



(A)


y = -x
y = x
B
1
-1
-π
π
2π
y = sin x
y = csc x


C
-π/2
π/2
3π/2
y = tan x

Formulas

$P = (a, b)$

$a^2 + b^2 = 1$

$\cos^2 x + \sin^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

Others

$A = B = \theta$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \sin B \cos A$

Double

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin 2\theta = 2 \sin\theta \cos\theta$

half-angle

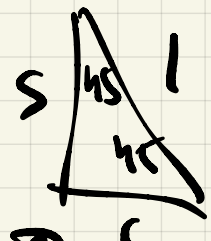
$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

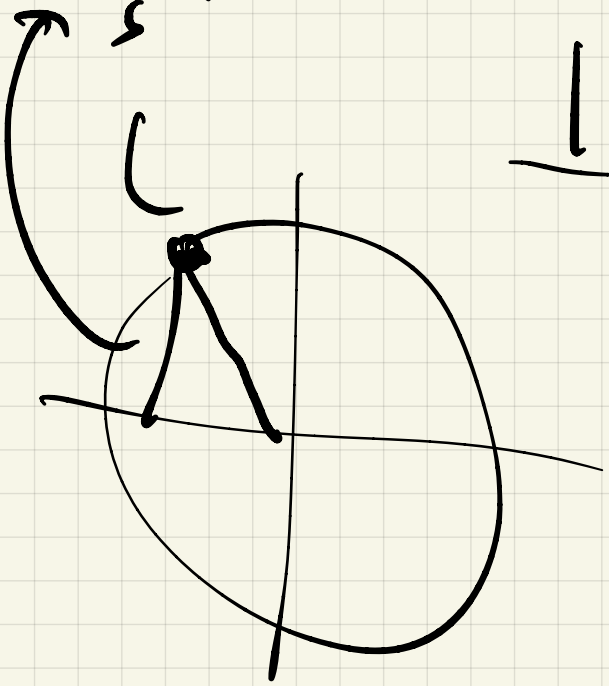
$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$ ⇐

Ex Find $\sin^2\frac{3\pi}{8}$

$s^2+s^2=1$

$s=\frac{1}{\sqrt{2}}$





$$\sin^2\frac{3\pi}{8} = \frac{1-\cos 2\left(\frac{3\pi}{8}\right)}{2}$$

$$\frac{1-\cos\frac{6\pi}{8}}{2} = \frac{1-\cos 3\pi/4}{2}$$

$$\frac{1-\left(-\frac{1}{\sqrt{2}}\right)}{2} =$$

$$\frac{1+\frac{1}{\sqrt{2}}}{2} = \frac{\sqrt{2}+1}{2\sqrt{2}}$$

$$\therefore \sin\frac{3\pi}{8} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

Two more comments

Two inequalities:

$$-|\theta| \leq \sin\theta \leq |\theta|$$

$$-|\theta| \leq 1-\cos\theta \leq |\theta|$$