Calc I:

Contact info:

Text

TI-83

Calculator TI-84

Grades

- Quiz
- HW
- Exams
- Final

Weekly Planner:

Overview:

What is Calculus?

Newton: tool for physics

Example Instantaneous velocity

A car accelerates down a track, it's distance is recorded

table

t (seconds)	0	1	2	3	4	5
d (feet)	0	8.8	35.2	79.2	140.8	220

(A) What is average speed over the 5 seconds?

Precalc

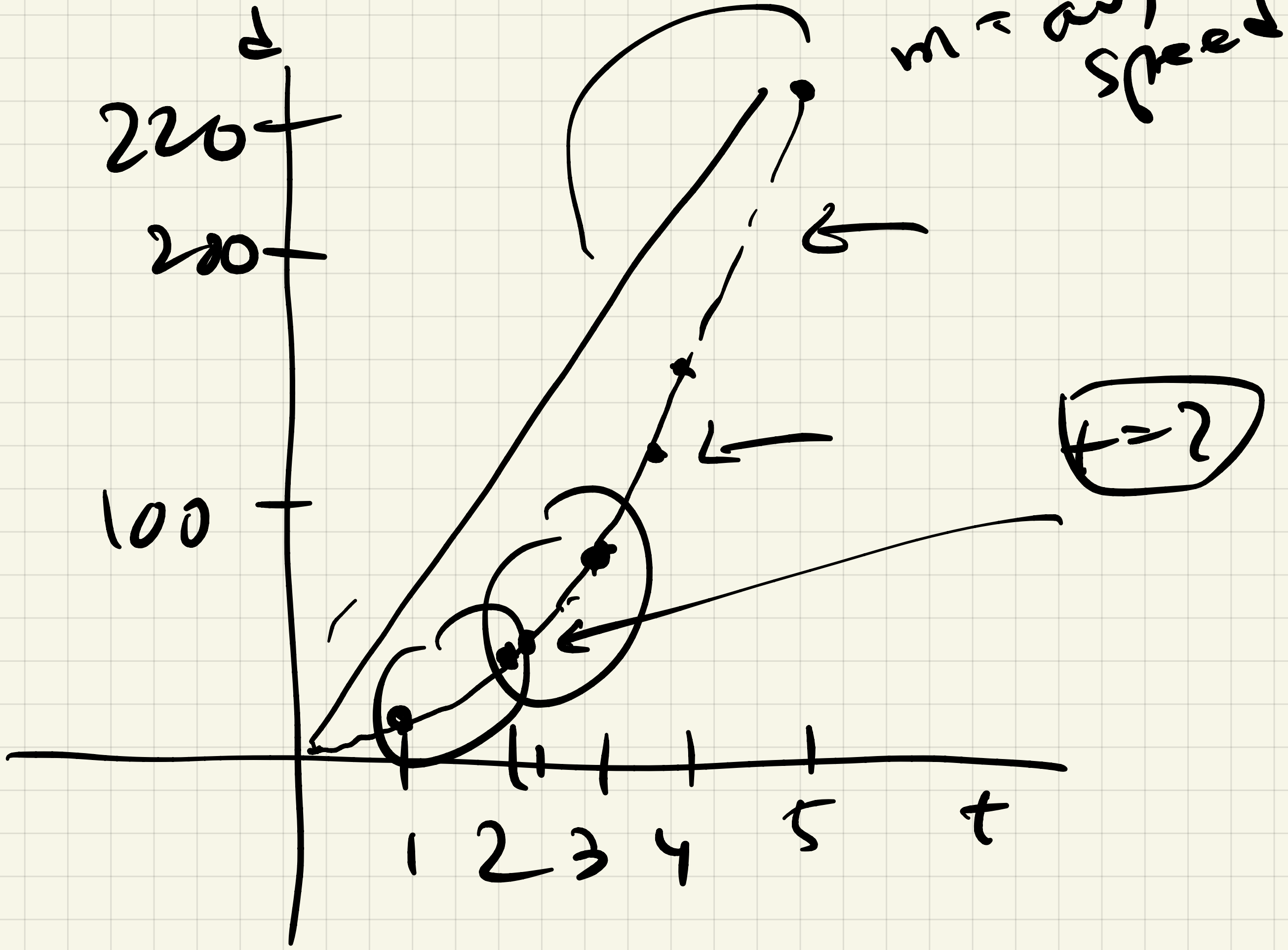
$$\frac{220}{5} = 44 \text{ ft}/\text{sec} = 30 \text{ mph}$$

(B) What is the exact speed after 2 seconds?

Calc Harder

Can we say anything?

Graph



Yes

① avg speed $[1,2] = \frac{35.2 - 8.8}{2-1} =$ 26.4 ft/sec

② avg speed $[2,3] = \frac{79.2 - 35.2}{3-2} =$ 44 ft/sec

$26.4 \text{ ft/sec} < \text{Exact speed} < 44 \text{ ft/sec}$

More data: At time $t = 2.1$, distance is 38.808 ft.

3 avg speed $[2, 2.1] =$

$$\frac{38.808 - 35.2}{.1} = \frac{3.608}{.1} =$$

36.08 ft/sec.

$$\underline{26.4} < ES < \underline{36.08}$$

Here's all data:

$$\boxed{d = 8.8t^2}$$

Now can make truly

accurate estimate:

for $\Delta t > 0$,

$$\left(\begin{matrix}\text{avg speed} \\ [2-\Delta t, 2]\end{matrix}\right) < ES < \left(\begin{matrix}\text{avg} \\ \text{speed} \\ [2, 2+\Delta t]\end{matrix}\right)$$

$$\frac{\text{dist}}{\text{time}} \qquad\qquad \frac{\text{dist}}{\text{time}}$$

$$\frac{8.8 \cdot 2^2 - 8.8(2-\Delta t)^2}{\Delta t} < ES < \frac{8.8(2+\Delta t)^2 - 8.8 \cdot 2^2}{\Delta t}$$

LHS

$$= \frac{8.8\left(2^2 - (2-\Delta t)^2\right)}{\Delta t}$$

$$(2-\Delta t)^2 = 2^2 - 2\Delta t - 2\Delta t + \Delta t^2$$

$$\frac{8.8\left(\cancel{4}-\left(\cancel{4}-4\Delta t+\Delta t^2\right)\right)}{\Delta t}$$

$$\frac{8.8\left(\quad +4\Delta t-\Delta t^2\right)}{\Delta t} =$$

$$\frac{8.8\left(4\Delta t-(\Delta t)^2\right)}{\Delta t} = 8.8(4-\Delta t)$$

$$8.8(4-\Delta t) < ES < 8.8(4+\Delta t)$$

all $\Delta t > 0$

$$35.2-8.8\Delta t < ES < 35.2+8.8\Delta t$$

as $\Delta t \to 0$

LHS $\to 35.2$

RHS $\to 35.2$ so $ES = 35.2$ ft/sec

Further if know exact speed at each time, can estimate/compute distance traveled.

Course Outline:

1. Functions & Graphs (precalc)
2. Limits (last step in example)
3. Derivatives (exact speed)
4. Applications (max/min problems, graphs)
5. Integration (exact distance)

Ch1: Functions & graphs

Defn: A **function** f from a set D (**domain**) to a set Y (target) is a rule assigning each x in D to exactly one y in Y

$x \in D$

$y \in Y$

Notation $y = f(x)$

Ex1 $D = \mathbb{R}$, $Y = \mathbb{R}$

$$f(x) = \begin{cases} 2 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

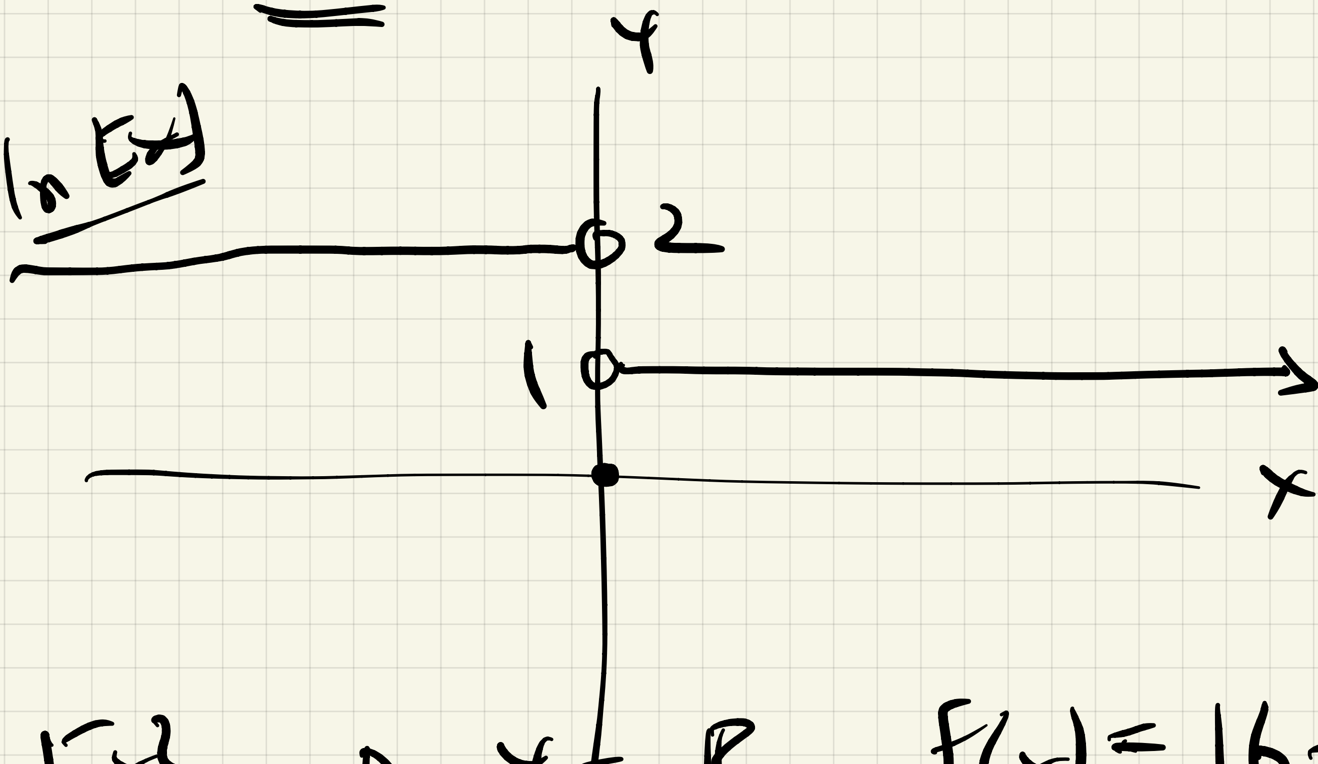
$f(-5000) = 2$ $\quad f(100) = 1$

$f(0) = 0$

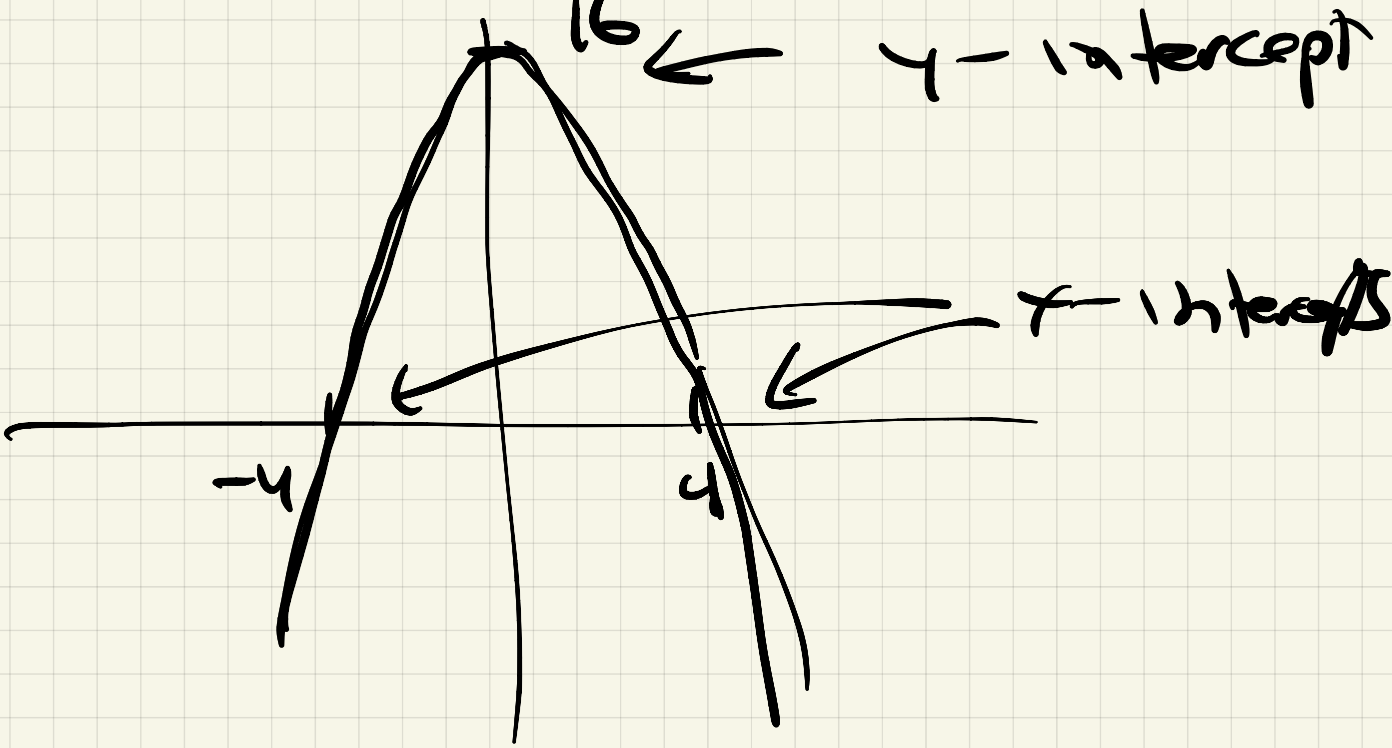
The graph of f is

$\{(x,y) \mid y = f(x)\}$ in $\mathbb{R}^2$

In [x]



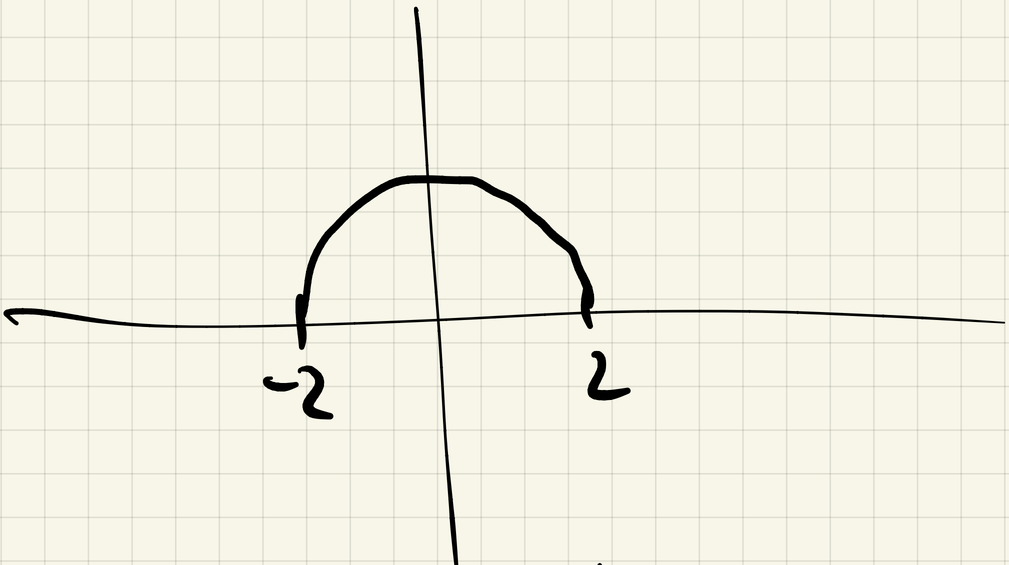
Ex2 $D = y = \mathbb{R}$ $f(x) = 16 - x^2$



Ex3 $f(x) = \sqrt{4-x^2}$

$\cancel{D = \mathbb{R}}$ $\quad D = [-2, 2]$

$\{x : f(x) \text{ defined}\}$



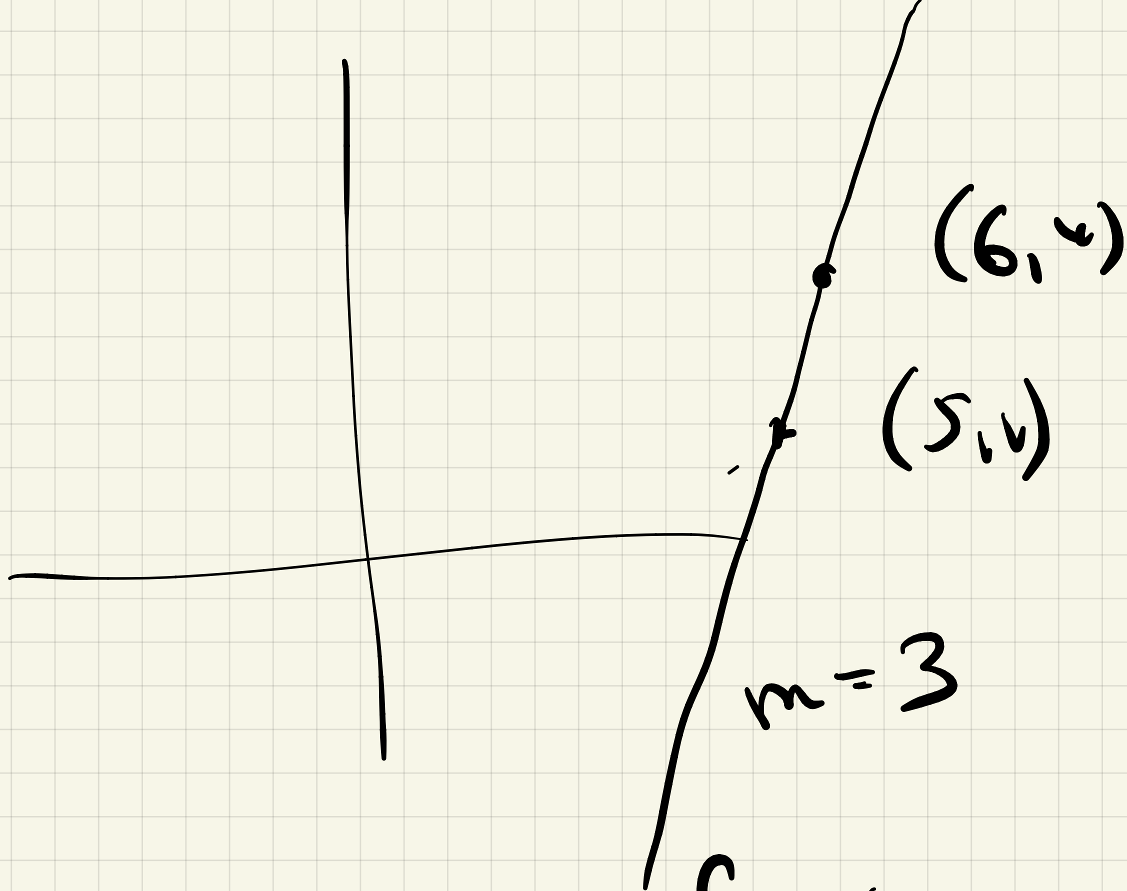
Straight lines:

The slope of the line between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$

is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Ex 1

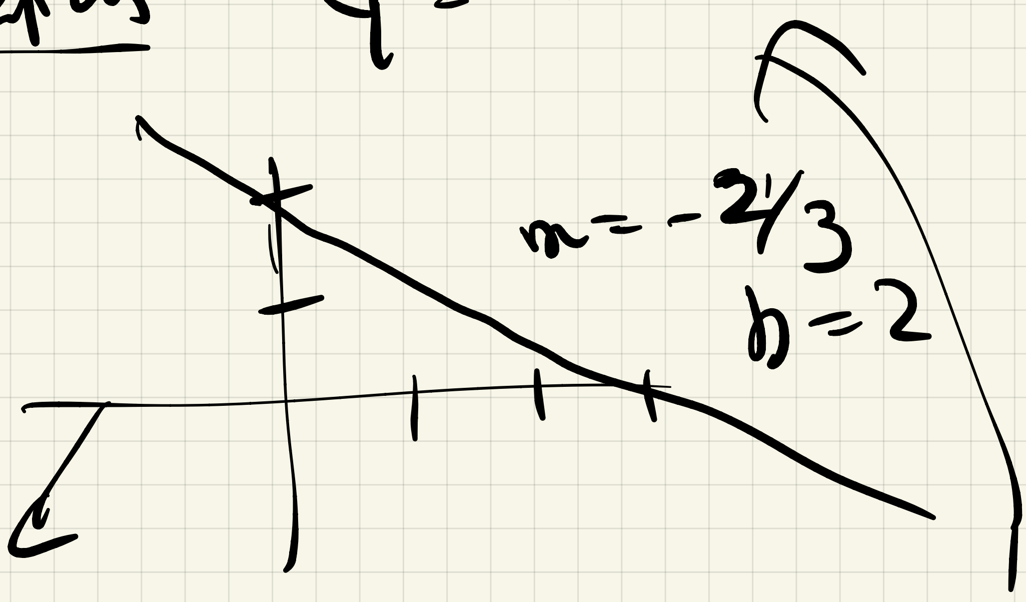


Two equations for lines

(A) If a line has slope m and y-int is b,

equation: $y = mx + b$

Ex 2



$y = 2 - \frac{2}{3}x$

$\boxed{\text{slope-int form}}$

B $\boxed{\text{point-slope form}}$

If slope is m

point on line is $P = (x_1, y_1)$

equation: $y - y_1 = m(x - x_1)$

In Ex1, $P = (6, 4)$

$m = 3$

$y - 4 = 3(x - 6)$

$y - 4 = 3x - 18$

$$\boxed{y = 3x - 14}$$

Slopes : //

① Parallel lines have same slope

$\perp$

② Perpendicular lines

$m_1 m_2 = -1$

$m_2 = \dfrac{-1}{m_1}$

Ex3 If L is $y = 3x - 14$

ⓐ Find eqn. of line // to L through $(2,1)$

$m = 3$

$y - 1 = 3(x-2) = 3x - 6$

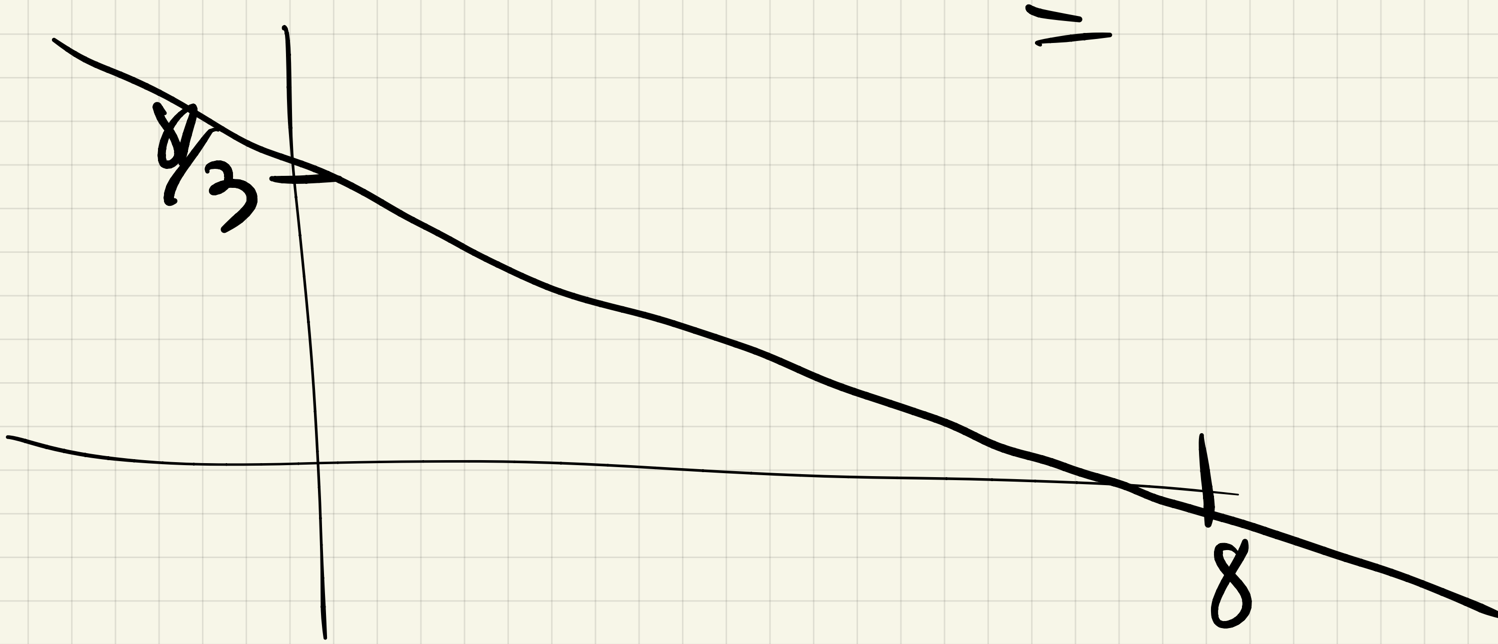
$y = 3x - 5$

ⓑ Find line $\perp$ to L through $(8,0)$

$m = -1/3$

$$y - 0 = -\frac{1}{3}(x-8)$$

$$y = -\frac{1}{3}x + \underline{\underline{8/3}}$$

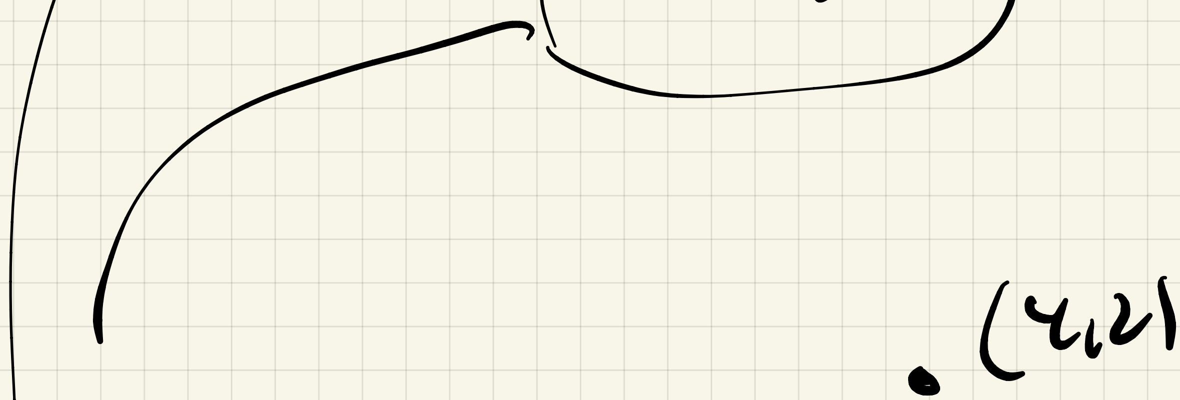


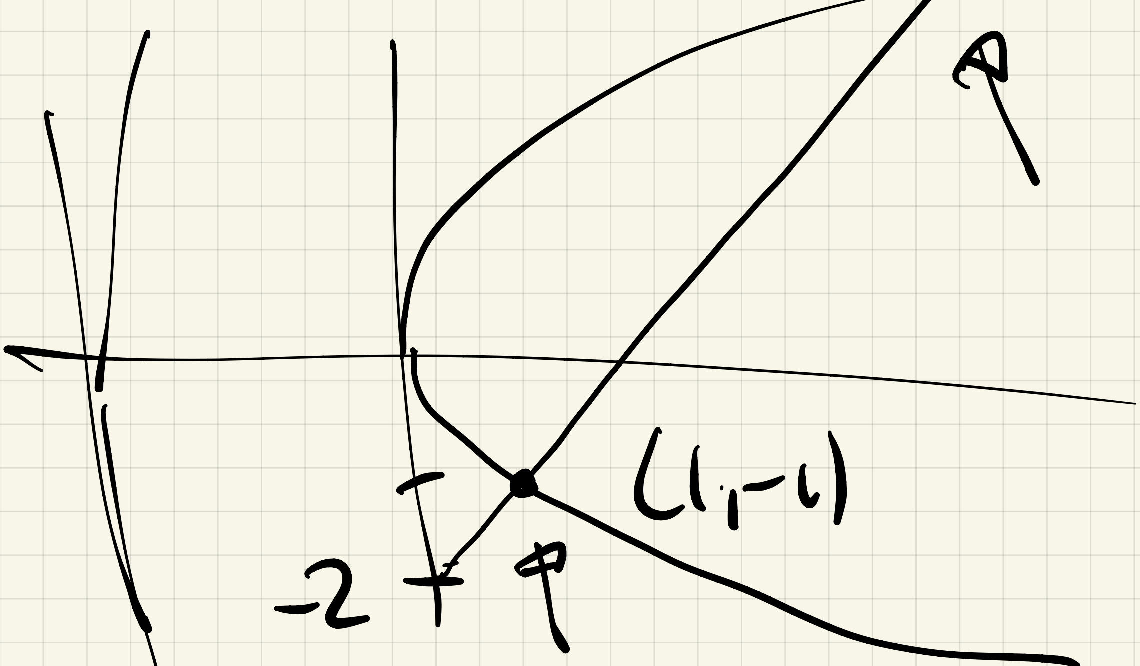
__Ex 4__ Where does the line

$\boxed{y = x-2}$ ←

intersect the curve

$x = y^2$??





$$\boxed{y = x-2}$$

$$x = y^2$$

$$\Rightarrow x = (x-2)^2$$

$$x = x^2-4x+4$$

$$\boxed{\begin{aligned} 0 &= x^2-5x+4 \\ &(x-1)(x-4) \end{aligned}}$$

$x^2-5x-800$

$x = 1, 4$

$y = -1 \quad 2$

$(1,-1) \qquad (4,2)$