2/9/Calc1 Exam 1 Thursday

Last time
$$\begin{cases} \lim_{x \to c} f(x) = +\infty / -\infty \\ \lim_{x \to +\infty/-\infty} f(x) = L \end{cases}$$

2.6

§3.1 The slope of the curve $y = f(x)$ at $P = (x_0, f(x_0))$

is
$$\boxed{m} = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

"Difference quotient"

Tangent line to $y = f(x)$ at P

is the line through P with slope m

[Ex] Find the tangent line to $y = \dfrac{x}{x-5}$ $f(x)$ at $P = (2, -2/3)$

$x_0 = 2$

Need slope

$$m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} =$$

$$= \lim_{h \to 0} \frac{\boxed{f(2+h)} - f(2)}{h} =$$

0/0

$$= \lim_{h \to 0} \frac{\frac{2+h}{(2+h)-5} - \left(-\frac{2}{3}\right)}{h}$$

$$\lim_{h \to 0} \frac{\boxed{\frac{3}{3}} \frac{2+h}{h-3} + \frac{2}{3}\frac{(h-3)}{(h-3)}}{h} =$$

$$\lim_{h \to 0} \frac{\frac{3(2+h) + 2(h-3)}{3(h-3)}}{h} =$$

$$\lim_{h\to 0} \frac{\cancel{6}+3h+2h-\cancel{6}}{h\cdot 3(h-3)} =$$

$$\lim_{h\to 0} \frac{5\cancel{h}}{\cancel{h}\cdot 3\cdot(h-3)} =$$

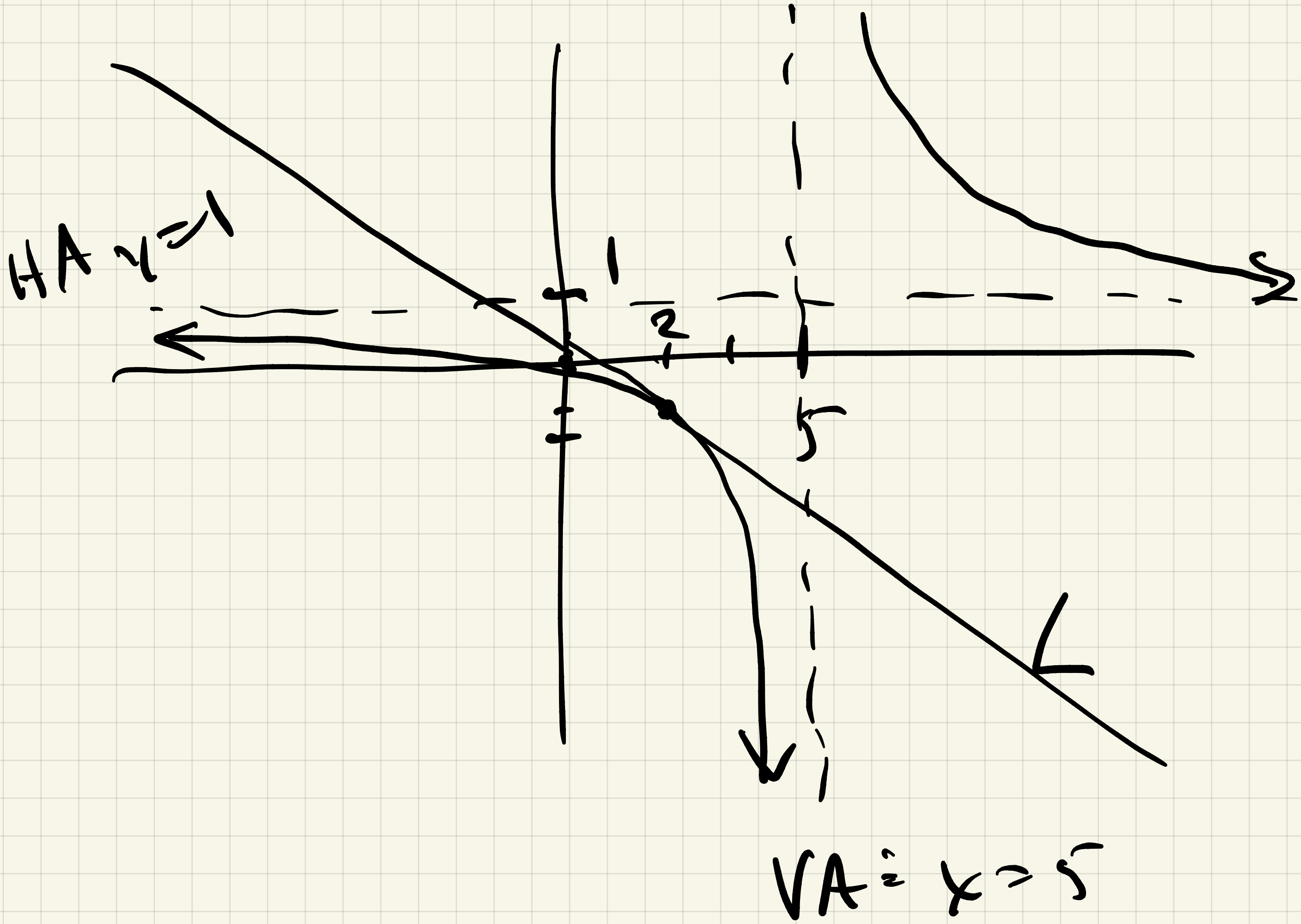
$$\lim_{h\to 0} \frac{5}{3(h-3)} = -5/9 = m$$

<u>Tangent line</u> $\qquad P = (2, -2/3)$

$$y - \overset{+2/3}{\left(-2/3\right)} = -5/9\,(x-2)$$

$$y = -5/9\,x + 10/9 - 2/3$$

$$= -5/9\,x + 4/9$$



Other language: m = slope of curve is also called the derivative of f at x_0

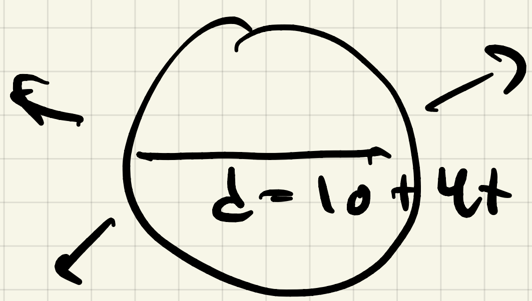
denoted $f'(x_0) = m$

Importance in Physics/ Applications

If t = time and $f(t)$ represents a physical quantity (position of a moving, volume, areas weight), then

$f'(t_0)$ is the <u>rate of change</u> of $f(t)$ with respect to time.

<u>Ex1</u> If a circle has diameter $d = 10 + 4t$ inches for $t \geq 0$ (t in seconds)



(a) How fast is radius of circle increasing at $t = 10$ sec?

(b) How fast is area of circle increasing?

(a) $r(t)$ = radius at time t,

(Find $r'(10) = ??$)

$$r(t) = \frac{d}{2} = \frac{10+4t}{2} = \boxed{5+2t}$$

$$r'(10) = \lim_{h\to 0} \frac{r(10+h) - r(10)}{h} =$$

$$\lim_{h\to 0} \frac{\cancel{5} + 2(\cancel{10}+h) - (\cancel{5} + 2(\cancel{10}))}{h}$$

$$= \lim_{h\to 0} \frac{2h}{h} = \lim_{h\to 0} 2 = 2 \text{ in/sec}$$

(b) **Area**:

$$A(t) = \pi (r(t))^2 =$$
$$= \pi (5+2t)^2$$

Want:

$$A'(10) = \lim_{h \to 0} \frac{\boxed{A(10+h)} - A(10)}{h}$$

$$= \lim_{h \to 0} \frac{\pi(5+2(10+h))^2 - \pi(5+2(10))^2}{h}$$

$$= \lim_{h \to 0} \frac{\pi(\cancel{25} + 20(\cancel{10}+h) + 4(10+h)^2) - \pi(\cancel{25} + \cancel{20(10)} + 4(10)^2)}{h}$$

$$= \lim_{h \to 0} \frac{\pi(20h + 4(\cancel{100} + 20h + h^2)) \cancel{- \pi 4 (100)}}{h}$$

$$= \lim_{h \to 0} \frac{\pi(20h + 80h + 4h^2)}{h}$$

$$= \lim_{h \to 0} \frac{\pi(20 + 80 + \overset{\to 0}{\cancel{4h}})}{1}$$

$$= \lim_{h \to 0} 100\pi \quad \text{inches/sec.}$$