2/8 | Calc I

Quiz 6 $0/0$

1. $\lim_{x \to 3} \frac{x^2-9}{x^2-x-6} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x+2)}$

$= \lim_{x \to 3} \frac{x+3}{x+2} = \frac{6}{5}$.

$(a-b)(a+b) = a^2-b^2$

$0/0$

2. $\lim_{x \to 3} \frac{\sqrt{x+6}-3}{x-3} \cdot \frac{(\sqrt{x+6}+3)}{(\sqrt{x+6}+3)}$

$\lim_{x \to 3} \frac{(\sqrt{x+6})^2 - 3^2}{(x-3)(\sqrt{x+6}+3)} =$

$\lim_{x \to 3} \frac{(x+6)-9}{(x-3)(\sqrt{x+6}+3)} =$

$\lim_{x \to 3} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x+6}+3)} =$

$\lim_{x \to 3} \frac{1}{\sqrt{x+6}+3} = \frac{1}{3+3} = \frac{1}{6}$

$$3.\quad \lim_{x\to -3} \frac{\sqrt{x+6}-3}{(x-3)} = \frac{\sqrt{3}-3}{-6}$$

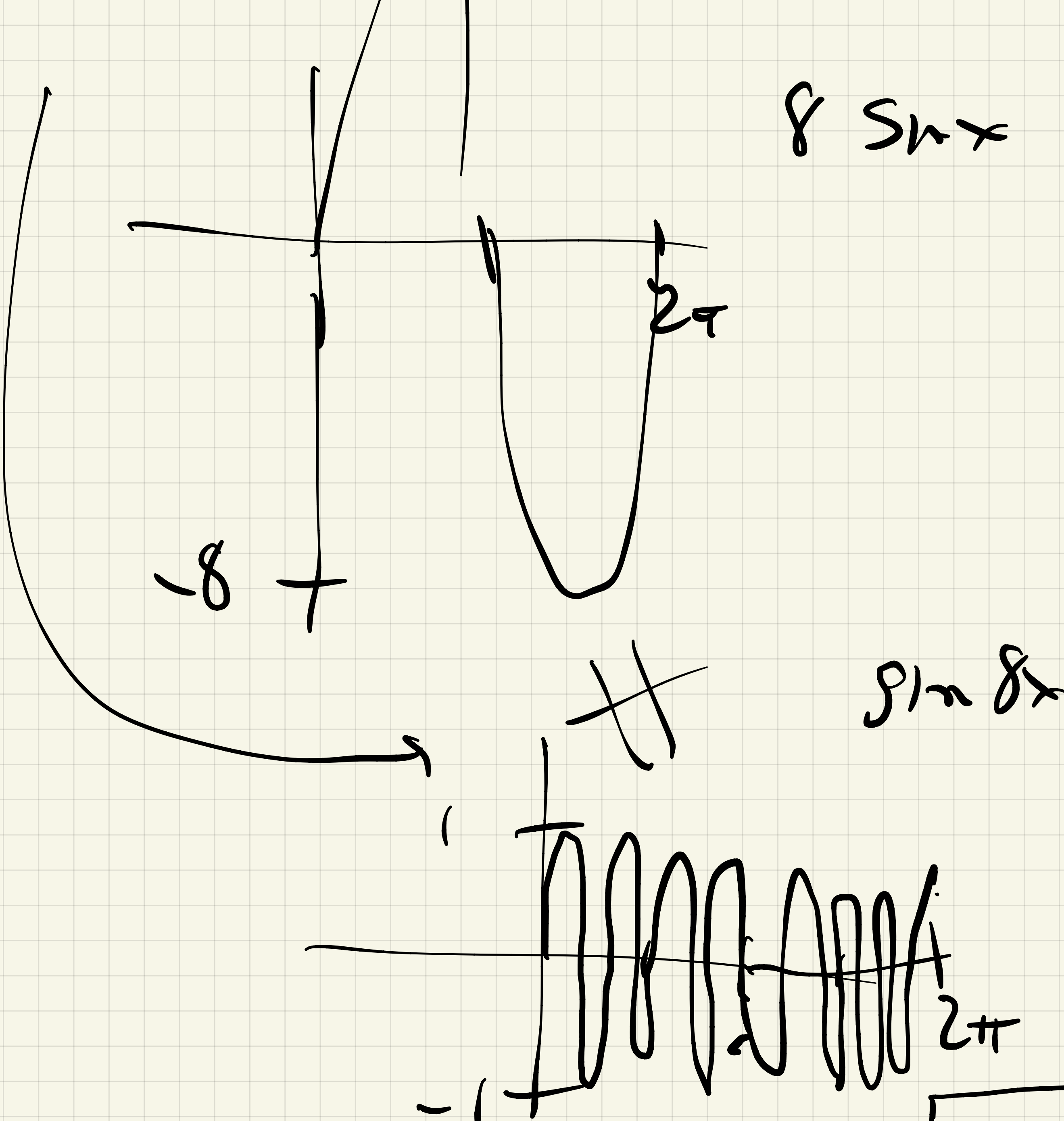
~~4~~ plug-in !

$$4.\quad \lim_{x\to 0} \frac{\tan 8x}{2x} =$$

$$\lim_{x\to 0} \frac{\frac{\sin 8x}{\cos 8x}}{2x} =$$

$$\lim_{x\to 0} \frac{\sin 8x}{2x \cos 8x}$$

$$\sin \boxed{8x} \neq \boxed{8} \sin x$$



$$\lim_{x \to 0} \frac{\sin 8x}{2x \cdot \boxed{\cos 8x}}$$

$$\boxed{\lim_{u \to 0} \frac{\sin u}{u} = 1}$$

use this!

$$\lim_{x\to 0} \frac{4}{1} \boxed{\frac{\sin 8x}{8x}} \frac{1}{\cos 8x}$$

$$4 \cdot 1 \cdot \frac{1}{1} = 4.$$

§2.6 Limits involving infinity

Ex) What $\lim_{x\to 0} \frac{1}{x^2}$ DNE

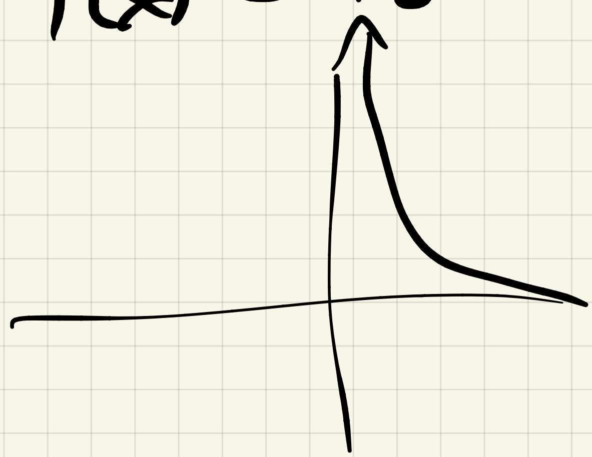
but: it fails to exist in a particular way:

as $x \to 0$, $\frac{1}{x^2}$ increases without bound

Defn: $\lim_{x\to c} f(x) = +\infty$ means that $f(x)$ grows without bound as x approaches c.

Similarly for $\lim_{x\to c^-} f(x) = +\infty$

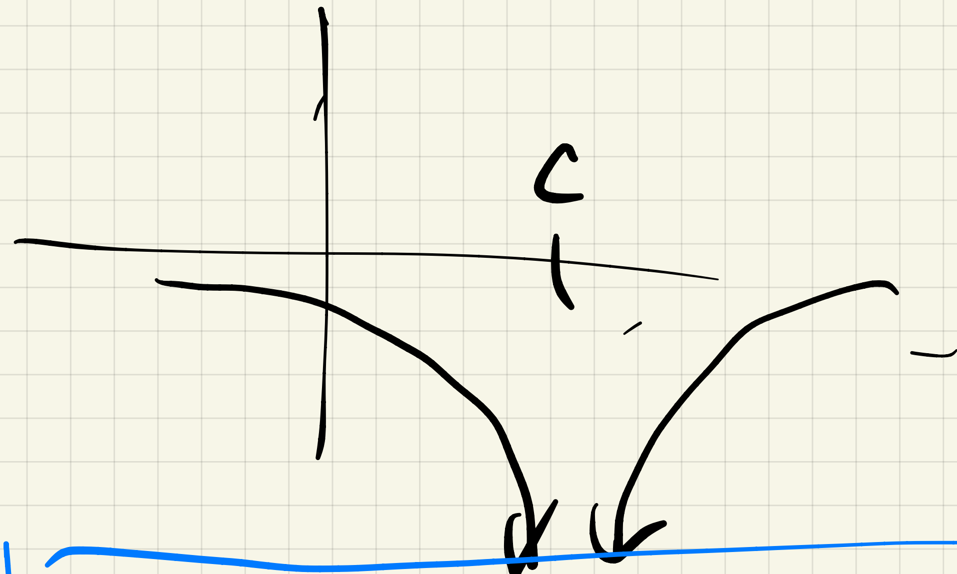
$\lim_{x\to c^+} f(x) = +\infty$



Also

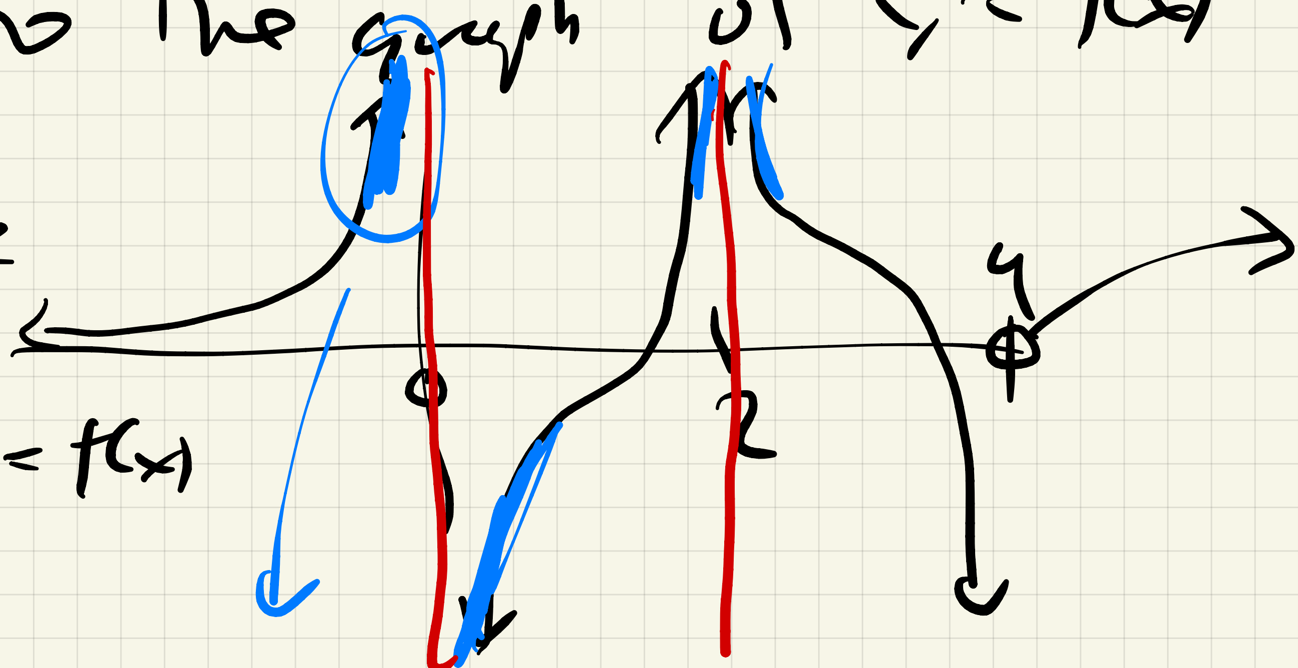
$$\left.\begin{array}{l} \lim_{x\to c} f(x) = -\infty \\ \lim_{x\to c^-} f(x) = -\infty \\ \lim_{x\to c^+} f(x) = -\infty \end{array}\right\} \text{Similar except}$$

f(x) get large without, but f(x) negative



In any of these 6 cases, we'll say that $x=c$ is a vertical asymptote to the graph of $y=f(x)$

Ex 2



$$\lim_{x\to 0^-} f(x) = +\infty$$

$$\lim_{x\to 0^+} f(x) = -\infty$$

$c=0$

$x=0$ is vertical asymptote (V.A.)

$$\lim_{x\to 0} f(x) = \text{DNE}$$

$$\lim_{x\to 2^-} f(x) = +\infty$$

$$\lim_{x\to 2^+} f(x) = +\infty$$

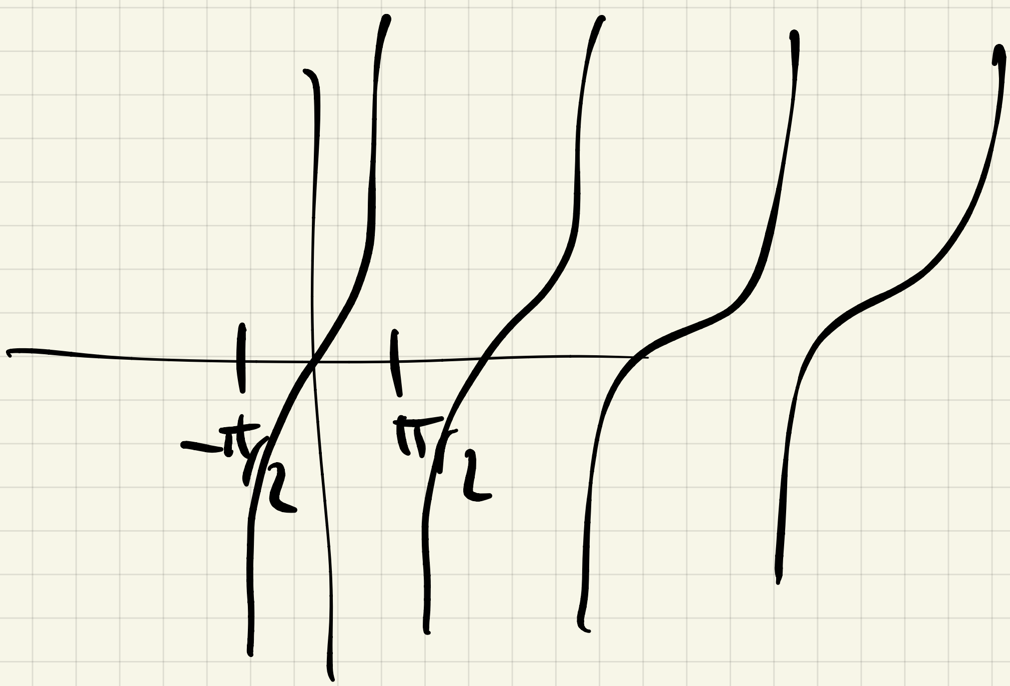
$$\lim_{x\to 2} f(x) = +\infty$$

$x=2$ is VA.

$$\lim_{x\to 4^-} f(x) = -\infty, \text{ so}$$

$\boxed{x=4 \text{ is V.A.}}$

Ex3 $y = \tan x$



many V.A.S.

Typical situation

$$f(x) = \frac{g(x)}{h(x)}$$ where

g, h continuous at $x=c$,

$g(c) \neq 0$, $h(c) = 0$

$\Downarrow$

$y=f(x)$ has V.A. at $x=c$

Ex 3 Find V.A.s

(a) $f(x) = \dfrac{x^2 - x}{x^4 - 5x^2 + 4} = \dfrac{x(x-1)}{(x-2)(x+2)(x-1)(x+1)}$

$x^4 - 5x^2 + 4 = (x^2)^2 - 5(x^2) + 4$

$y^2 - 5y + 4$

$(y-4)(y-1)$

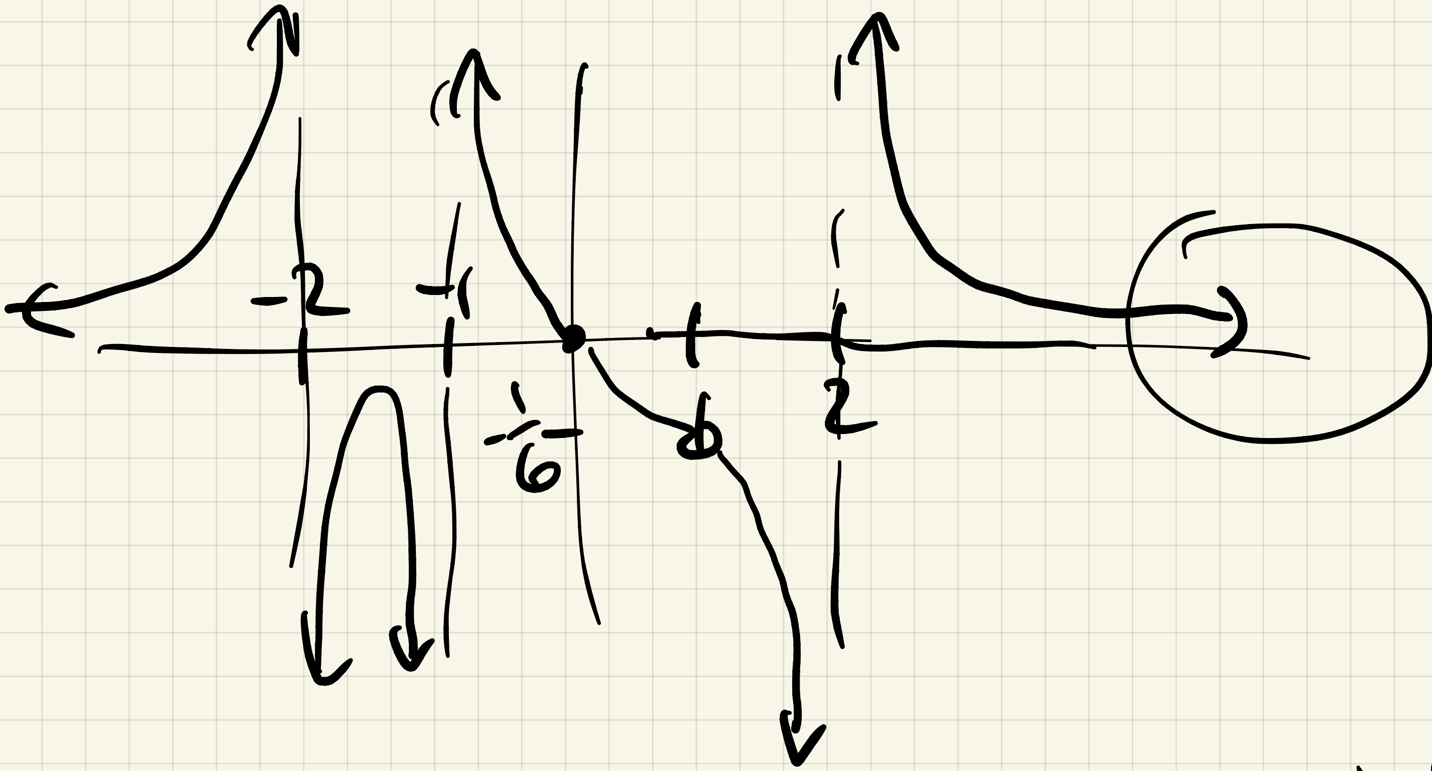
$(x^2-4)(x^2-1)$

$\boxed{x = 2,\ x = -2,\ x = -1}$

V.A.

$x = 1$: $\lim_{x \to 1} \dfrac{x\cancel{(x-1)}}{(x-2)(x+2)\cancel{(x-1)}(x+1)} = \dfrac{1}{-6}$

removable discontinuity



(b) $f(x) = \dfrac{x}{\sin x}$ candidates

ok, $\sin x = 0$ for $x = 0, \pm\pi$
$\pm 2\pi$.

$\lim\limits_{x \to 0} \dfrac{x}{\sin x} = 1$, so $x = 0$ ~~not~~ V.A.

but rest are, so

V.A. $\pm\pi, \pm 2\pi, \ldots$

<u>Defn</u> $\lim_{x \to +\infty} f(x) = L$ if we can make $f(x)$ as close to L as we like by choosing $x > 0$ large enough

$\lim_{x \to -\infty} f(x) = L$ if _ ___

_ __ ___ _________

__ ___ $x < 0$

In either case, $y = L$ is <u>horizontal asymptote</u> (H.A.)

<u>trick</u>: divide by (largest power of x

(a) $\lim_{x \to +\infty} \dfrac{x^2 - x}{x^4 - 5x^2 + 4} =$ indeterm.

$$= \lim_{x \to +\infty} \frac{\frac{1}{x^2} - \frac{1}{x^3}}{1 - 5/x^2 + 4/x^4} = \frac{0}{1} = 0$$

as $x \to +\infty$, $\frac{1}{x^2} \to 0$, $\frac{1}{x^3} \to 0$

$5/x^2 \to 0$ $\quad 4/x^4 \to 0$

(b) $\lim_{x \to -\infty} \dfrac{x^2 - x}{x^4 - 5x^2 + 4} = \dfrac{0}{1}$

same

(c) $\lim_{x \to +\infty} \dfrac{9x^4}{3x^4 + 5x^2 + 2} =$

$$\lim_{x \to +\infty} \frac{9}{3 + 5/x^2 + 2/x^4} = \frac{9}{3} = 3$$

(d) $\displaystyle\lim_{x\to-\infty} \frac{9x^4}{3x^4+5x^2+2} = 3$ $\quad 0$

(e) $\displaystyle\lim_{x\to\infty} \frac{3+\sqrt{x}}{5-6\sqrt{x}} =$

$$\lim_{x\to\infty} \frac{3/\sqrt{x}+1}{5/\sqrt{x}-6} = -\frac{1}{6}$$

$$\lim_{x\to-\infty} \frac{3+\sqrt{x}}{5-6\sqrt{x}} = \text{DNE}$$

(f) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{5x^6+20}}{x^3+8} =$??

$$\lim_{x\to\infty} \frac{\boxed{\dfrac{\sqrt{5x^6+20}}{x^3}}}{1+8/x^3} =$$

$2\sqrt{5} \neq \sqrt{10} =$

$$\lim_{x\to\infty} \frac{\sqrt{\frac{5x^6+20}{x^6}}}{1+8/x^3} \qquad = (x^3)^2$$

$\sqrt{2^2\cdot 5}$

$\sqrt[4]{20}$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{5+20/x^6}{1}}}{1+8/x^3} = \sqrt{5}$$

(g) $$\lim_{x\to-\infty} \frac{\sqrt{8x^6+20}}{x^3+8} =$$

$$\lim_{x\to-\infty} \frac{\frac{\sqrt{5x^6+20}}{x^3}}{1+8/x^3} =$$

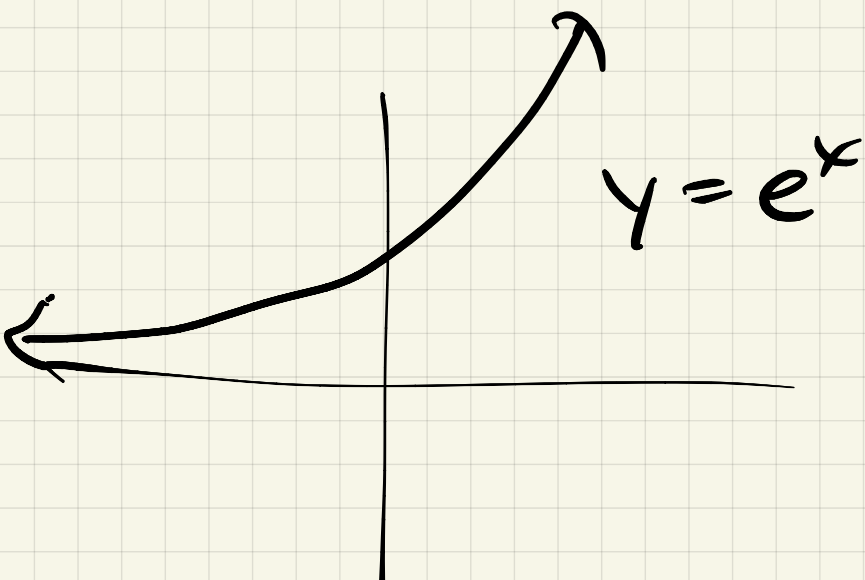
$$\lim_{x\to-\infty} \frac{-\sqrt{\frac{5x^6+20}{x^6}}}{1+8/x^3} = -\sqrt{5}$$

(h) $\displaystyle\lim_{x\to\infty} \frac{8-e^x}{2+3e^x} = \infty/\infty$

$$\lim_{x\to\infty} \frac{8/e^x - 1}{2/e^x + 3} = -\frac{1}{3}$$

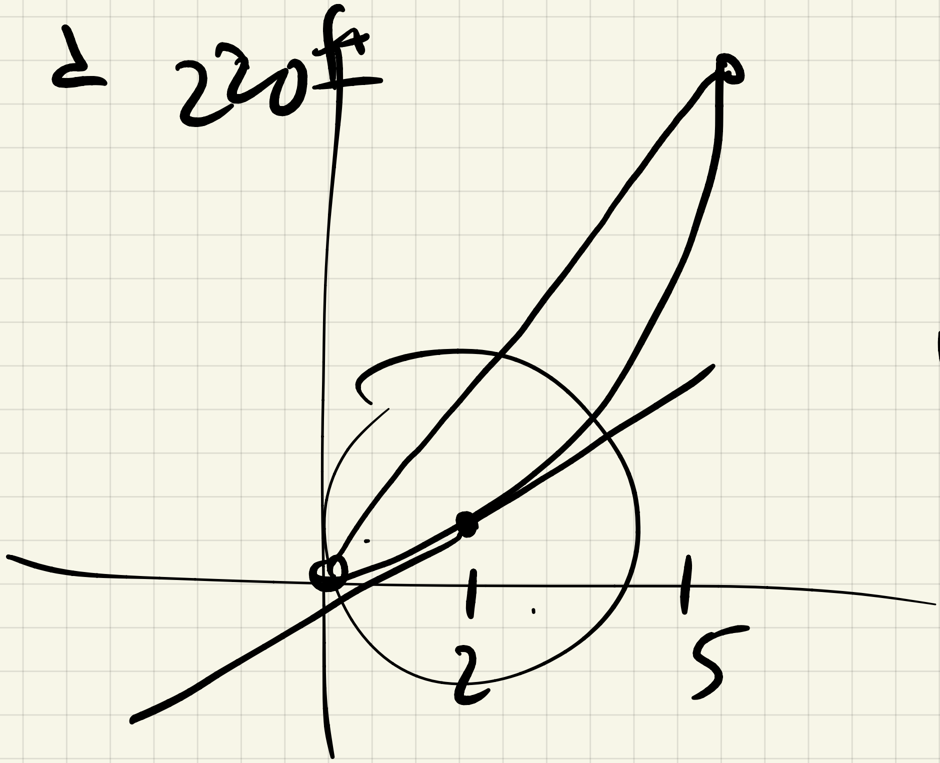
$$\lim_{x\to-\infty} \frac{8-e^x}{2+3e^x}$$

$e^x \to 0$ as $x \to -\infty$



$$\frac{8}{2} = 4.$$

§ 3.1 Recall the racing car from 1st ~~bon~~ of class



$d = 8.8t^2$

we made estimates for exact velocity at $t = 2$ seconds

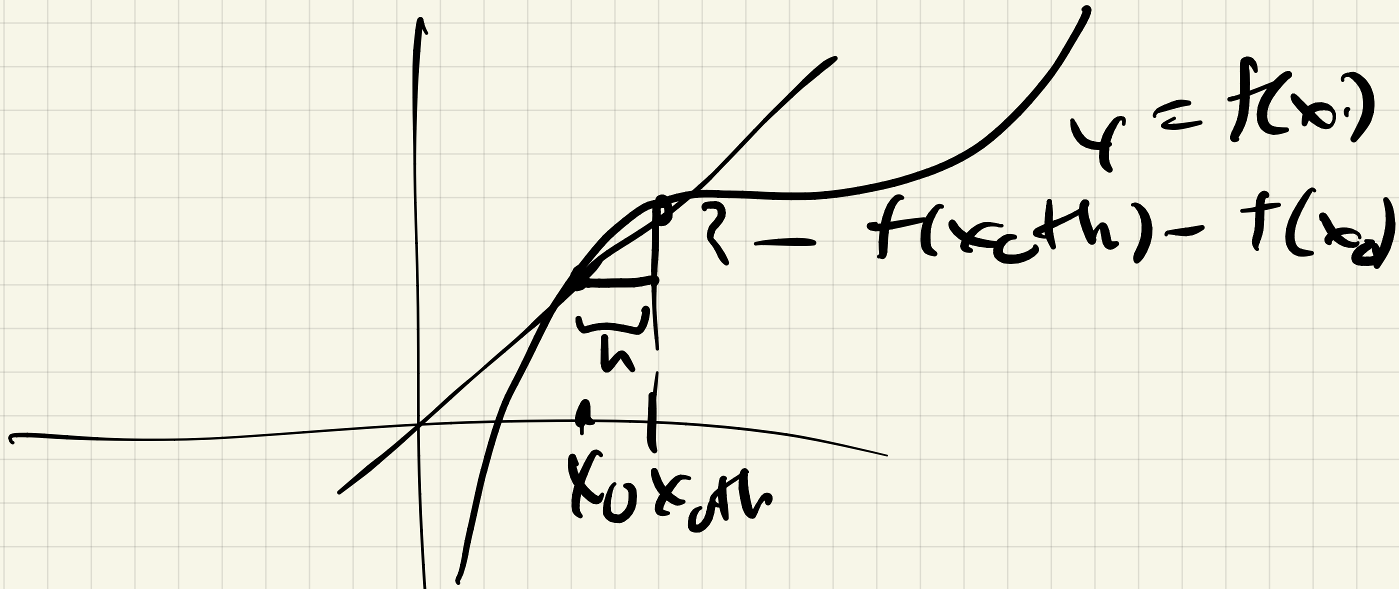
__This motivates__

__Defn__: The __slope__ of the curve $y = f(x)$ at $P = (x_0, f(x_0))$

is

$$\boxed{m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} \quad \text{(if it exists)}}$$

The tangent line to curve at P is the line thru P with slope m.



Ex 1 $\boxed{f(x) = 8.8x^2}$

$x_0 = 2$

The slope of $y = f(x)$ at $x_0 = 2$

$$m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} =$$

$$\lim_{h \to 0} \frac{8.8\boxed{(2+h)^2} - 8.8(2^2)}{h}$$

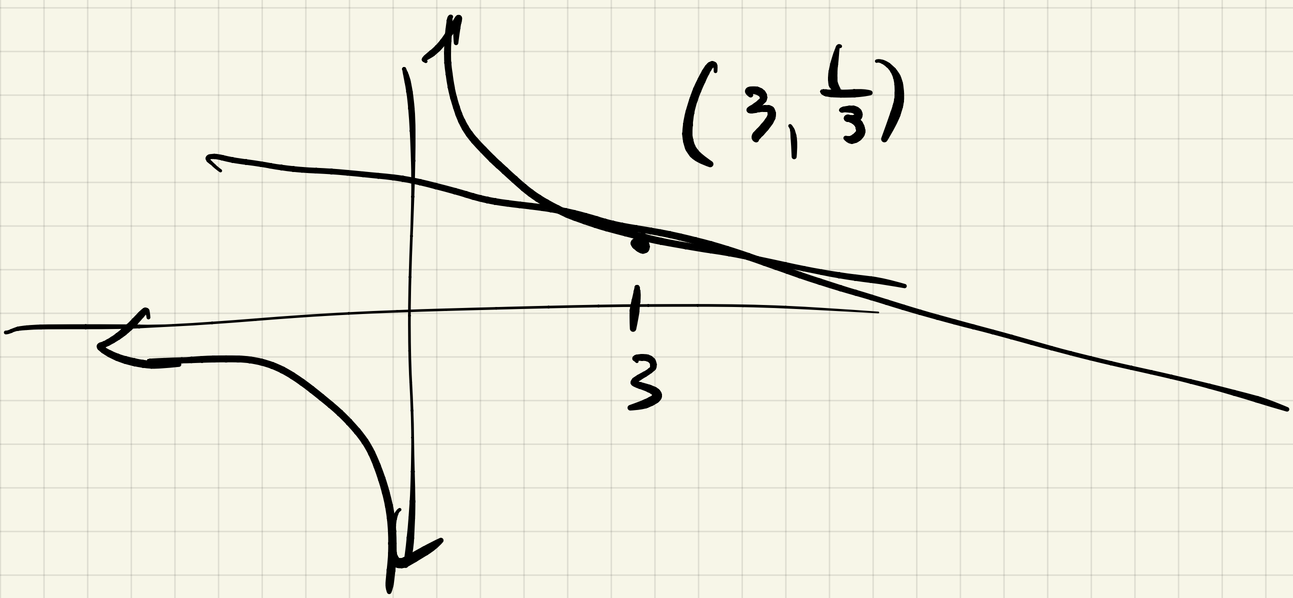
$$\lim_{h \to 0} \frac{8.8(\cancel{4}+4h+h^2) - \cancel{8.8(4)}}{h}$$

$$= \lim_{h \to 0} \frac{8.8(4h + h^2)}{h} =$$

$$\lim_{h \to 0} \frac{8.8(\cancel{h})(4+h)}{\cancel{h}} =$$

$$\lim_{h \to 0} \frac{8.8(4+h)}{1} = 8.8 \cdot 4 = \boxed{35.2}$$

<u>Ex 2</u> Find the slope of curve $y = \frac{1}{x} = f(x)$ at $x = 3$



$$\lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \frac{1}{x}$$

$$\lim_{h \to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} =$$

$$\lim_{h \to 0} \frac{\frac{3 - (3+h)}{3(3+h)}}{h} =$$

$$\lim_{h \to 0} \frac{-h}{h\,3 \cdot (3+h)} =$$

$$\lim_{h \to 0} \frac{-1}{3(3+h)} = -\frac{1}{9}$$