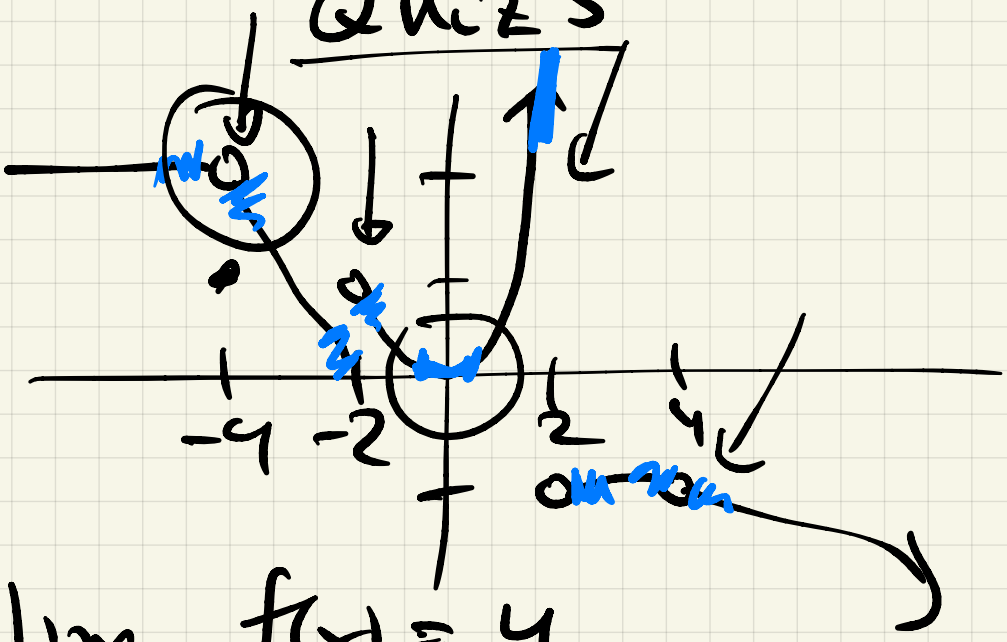
2/6/Calc1 HW 2.3, Qu 25

Qu 25



$\lim_{x \to -4} f(x) = 4$

$\lim_{x \to -2} f(x) = DNE$ — 2 diff values as $x \to -2$

$\lim_{x \to 0} f(x) = 0$

$\lim_{x \to 2} f(x) = DNE$ left $y \to \infty$

$\lim_{x \to 4} f(x) = -2$

Table $\lim_{x \to 0} (1-2x)^{1/x} = .1350$

I give values of x close to 0 from both sides

Remark

$$\lim_{x\to 0}(1-2x)^{1/x} = \lim$$

$$\lim_{x\to 0}(1+x)^{1/x} = e$$

$u = -2x$

$-\frac{1}{2}u = x$

$$\lim_{u\to 0}(1+u)^{1/-\frac{1}{2}u} =$$

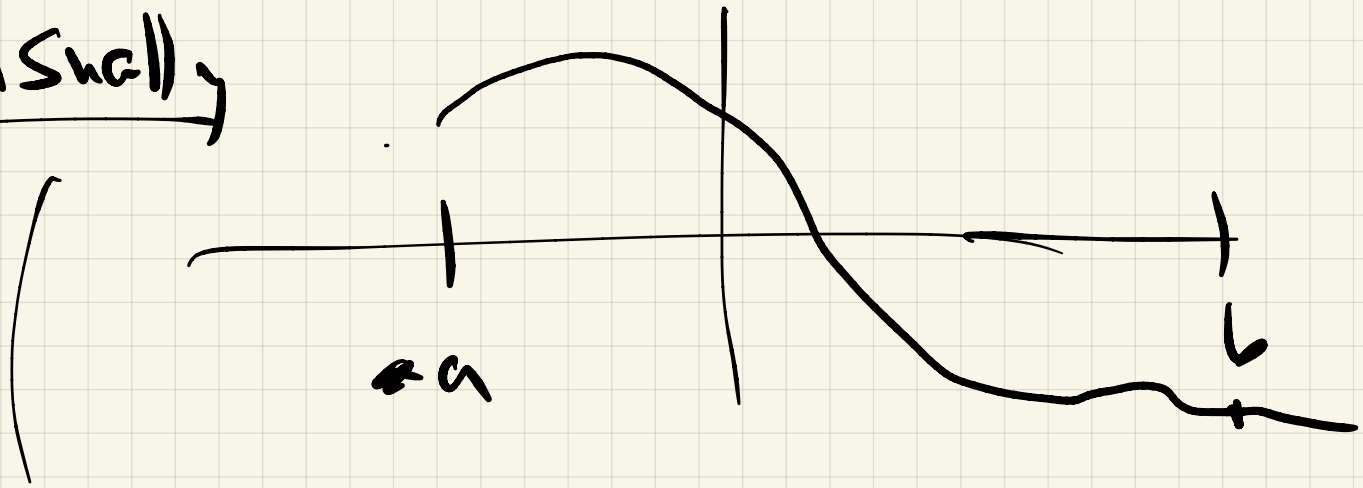
$$\lim_{u\to 0}(1+u)^{1/u(-2)} = \frac{1}{e^2}$$

Last time

Def f continuous at $x=c$ if $\lim_{x\to c} f(x) = f(c)$

f continuous on an interval if cont. at each pt in interval

Visually



No breaks / holes in graph.

Roughly speaking

Any function created from basic functions like polynomials, trig, inverse, exponential, logarithm

Ex)

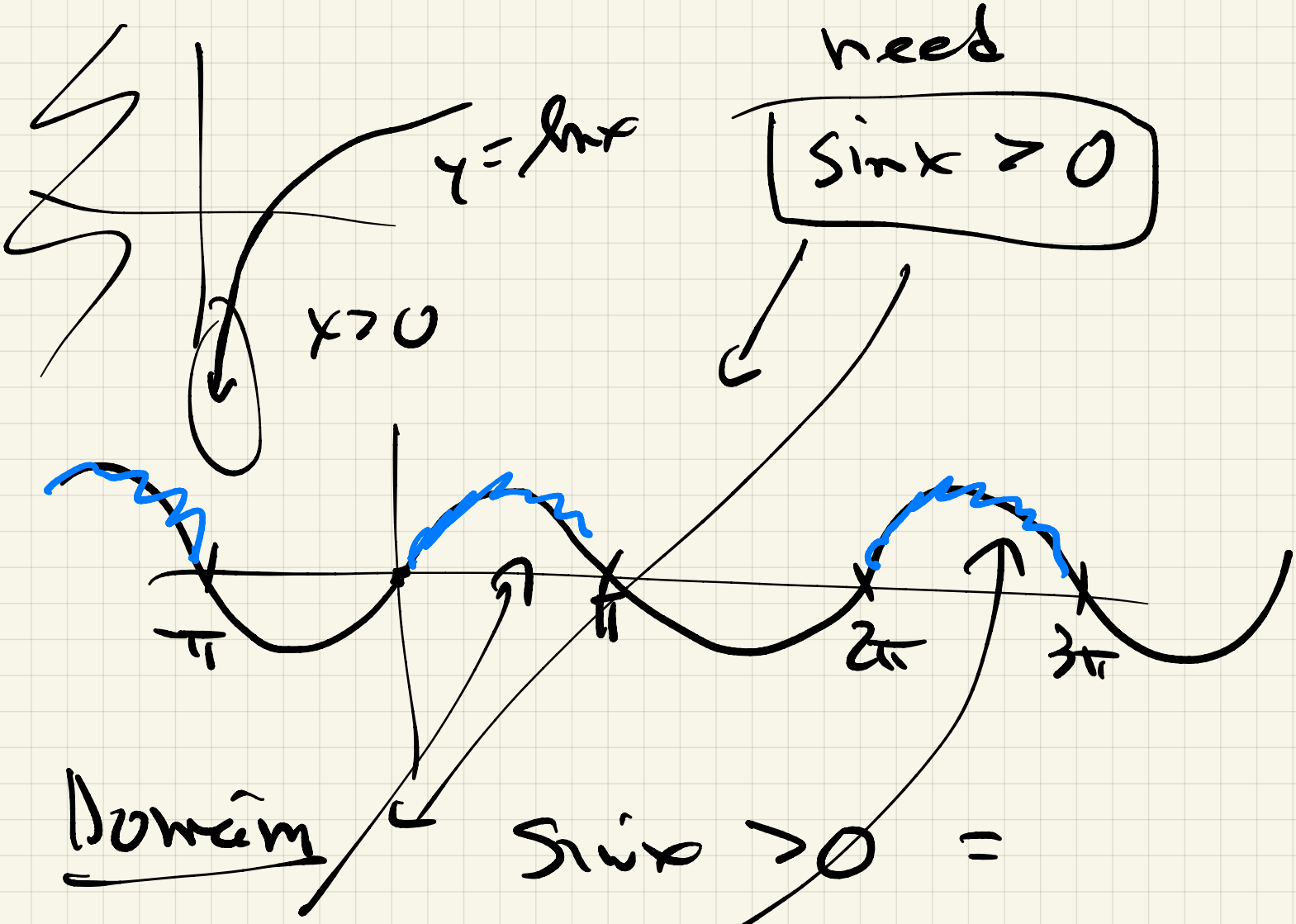
(a) $f(x) = \frac{\sqrt[3]{\sec^2 x + \ln(\tan x + 5) + x^3}}{\sqrt{1000 - x^2}}$

continuous on domain of

(messy to compute it)
(domain complicated)

(b) $y = f(x) = \ln(\sin x)$

need $\sin x > 0$



Domain $\sin x > 0$ =

$\cdots \cup (0, \pi) \cup (2\pi, 3\pi) \cup (4\pi, 5\pi) \cup \cdots$

$$\bigcup_{n \in \mathbb{Z}} (2n\pi, (2n+1)\pi)$$

(c) $f(x) = \dfrac{2x}{x^2-7x} = \boxed{\dfrac{2x}{x(x-7)}}$

Domain: $x \neq 0, 7$ i.e.,

$(-\infty, 0) \cup (0, 7) \cup (7, \infty)$

So $f(x)$ is discont at $c = 0, 7$

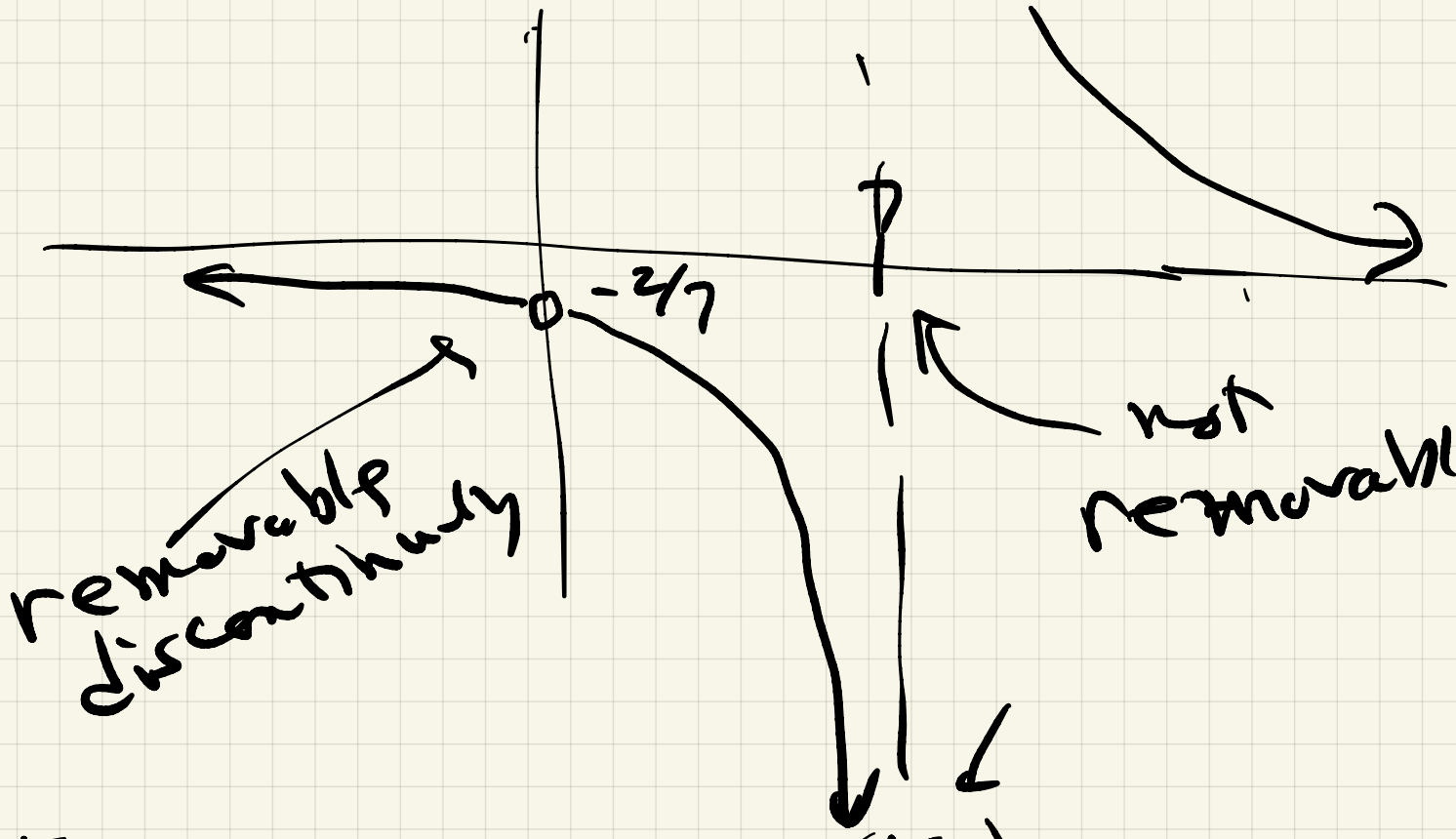
$c=0$: $\lim_{x \to 0} f(x) = \lim_{x \to 0} \dfrac{2x}{x(x-7)} =$

$\lim_{x \to 0} \dfrac{2}{x-7} = -2/7$

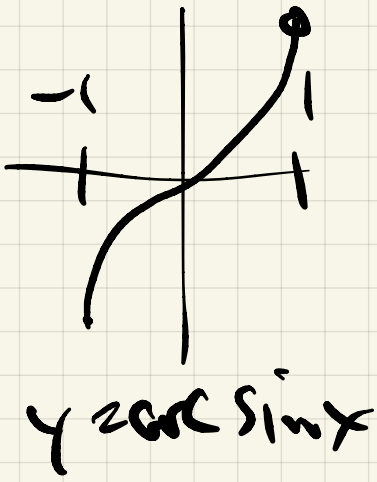
$c=7$ $\lim_{x \to 7} \dfrac{2}{x-7} = DNE$

A discontinuity is removable

if $\lim_{x \to c} f(x) =$ exists



Ex $f(x) = \arcsin\left(\frac{x}{20}\right) - 1$

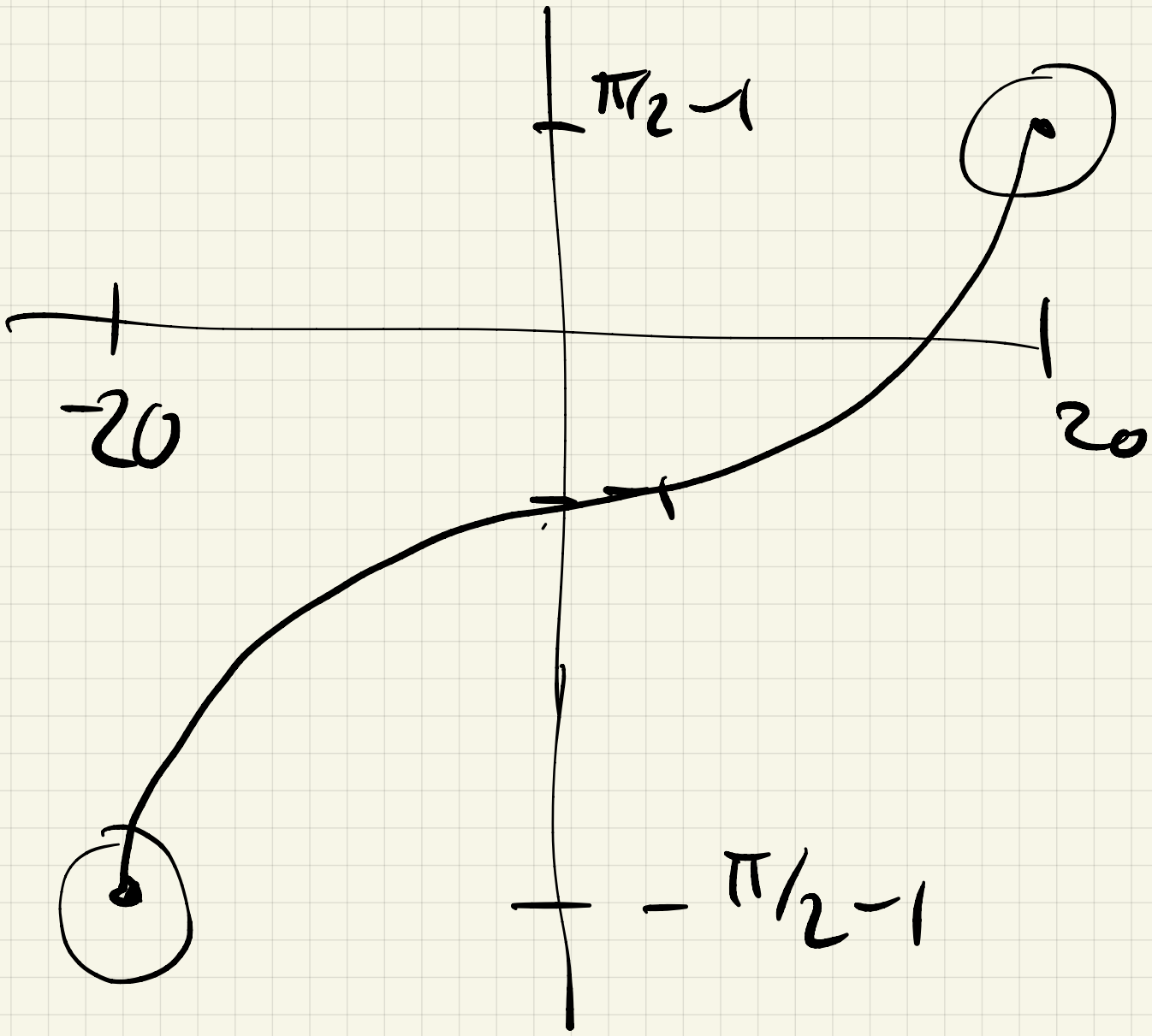


domain:

$$-1 \le \frac{x}{20} \le 1$$

$$-20 \le x \le 20$$

So continuous on $[-20, 20]$



Note We will consider $f(x)$ to be continuous on $[a,b]$ if $\lim_{x \to b^-} f(x) = f(b)$ and $\lim_{x \to a^+} f(x) = f(a)$

Remark: $\lim_{x \to c} f(x) = L$

$\Downarrow$

$\lim_{x \to c^+} f(x) = L$ and $\lim_{x \to c^-} f(x) = L$

so $f(x)$ is continuous at c if

$\lim_{x \to c^-} f(x)$

$f(c)$

$\lim_{x \to c^+} f(x)$

} all are equal

Ex 3 For which values c is $f(x)$ discontinuous at c?

$$f(x) = \begin{cases} \frac{1}{e^x - 5} & x > 0 \\ -\frac{1}{4} & x = 0 \end{cases}$$

$$\begin{cases} \dfrac{|x|}{x(x+4)} & x<0 \end{cases}$$

3 cases:

$c>0$ $\quad f(x)=\dfrac{1}{e^x-5}$

disc where $e^x-5=0 \Rightarrow$

$e^x=5 \Rightarrow$ **$x=\ln 5$**

$x=\ln(e^x)=\ln 5$

$c<0$ $\quad f(x)=\dfrac{|x|}{x(x+4)}=\dfrac{-x}{x(x+4)}$

$x<0$

disc at **$c=-4$**

$c=0$ $\quad \lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}\dfrac{1}{e^x-5}=-\dfrac{1}{4}$

$f(0) = -1/4$

$$\lim_{x \to 0^-} f(x) =$$

$$\lim_{x \to 0^-} \frac{-x}{x(x+4)} = \lim_{x \to 0^-} \frac{-1}{(x+4)} =$$

$$-\frac{1}{4}$$

cont at $c = 0$

Ans disc of $f(x)$ = $c = \ln 5$, $c = -4$

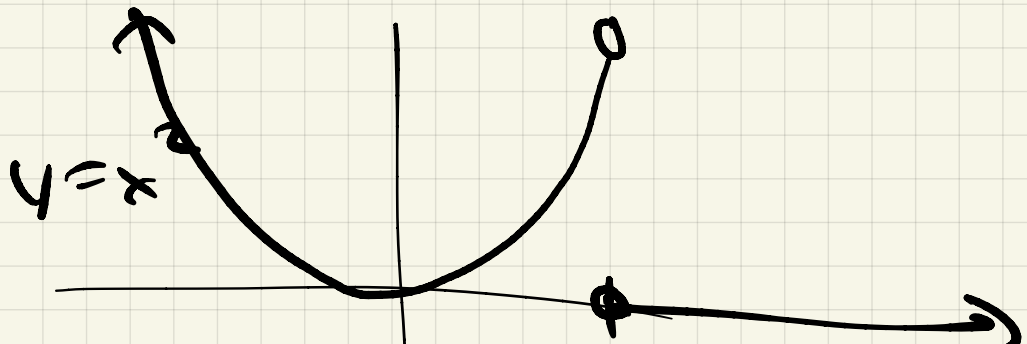
$$\{-4, \ln 5\}$$

Ex 3 For which number a is

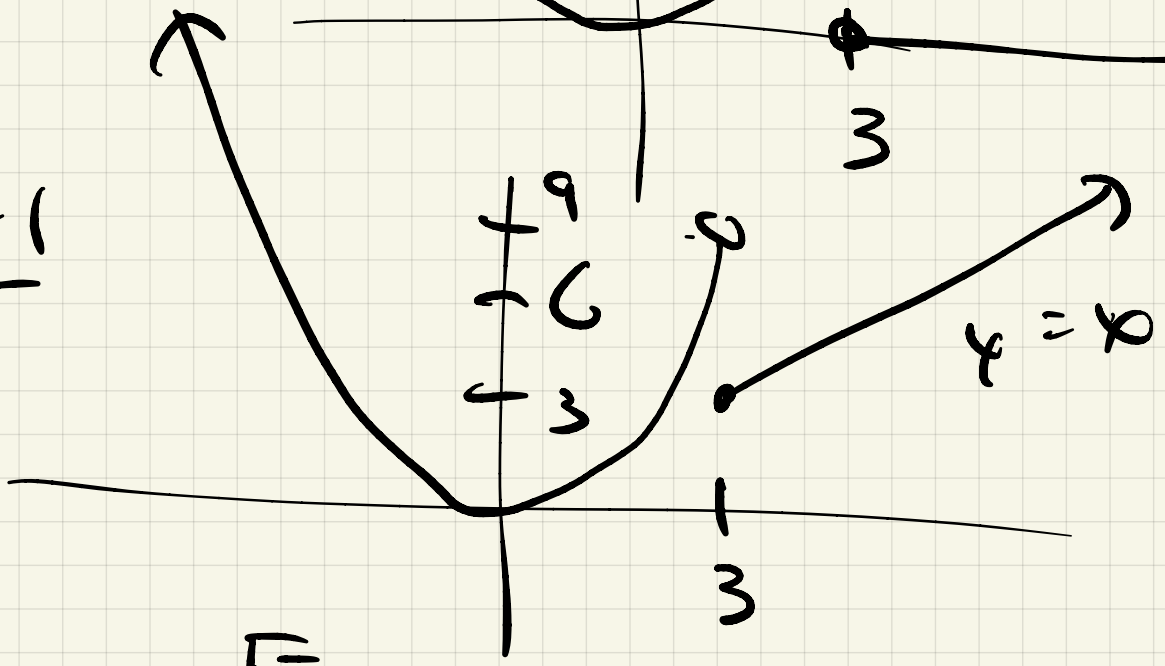
$$g(x) = \begin{cases} x^2 & x < 3 \\ a^2 x & x \geq 3 \end{cases}$$

continuous for all x in $\mathbb{R}$?

$a = 0$



$a = 1$



Note For any a, $g(x)$ continues on $(-\infty, 3)$

b/c $g = x^2$

$g(x)$ cont on $(3, \infty)$

b/c $g = a^2 x$

Just need to check $c = 3$

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x^2 = 9$$

$f(3) =$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} a^2 x = 3a^2$

need $7 = 3a^2$

$a^2 = 3$ $\quad a = \pm\sqrt{3}$

Ex 4 For what values a, b is $h(x)$ continuous for all x

$$h(x) = \begin{cases} \frac{\sin 7x}{x} & x < 0 \\ ax + b & 0 \le x \le 2 \\ x^2 & x > 2 \end{cases}$$

$h(x)$ continues at all

$c \neq 0, 2$

$c = 0$: $\lim_{x \to 0^-} h(x) = \lim_{x \to 0^-} \frac{\sin 7x}{x} = 7$

$h(0) = b$

$\lim_{x \to 0^+} h(x) = \lim_{x \to 0^+} ax+b = b$

$\boxed{b=7}$

$\underline{c=2}$: $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} ax+b = 2a+b$

$f(2) = 2a+b$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^2 = 4$

Need $\boxed{2a+b=4}$

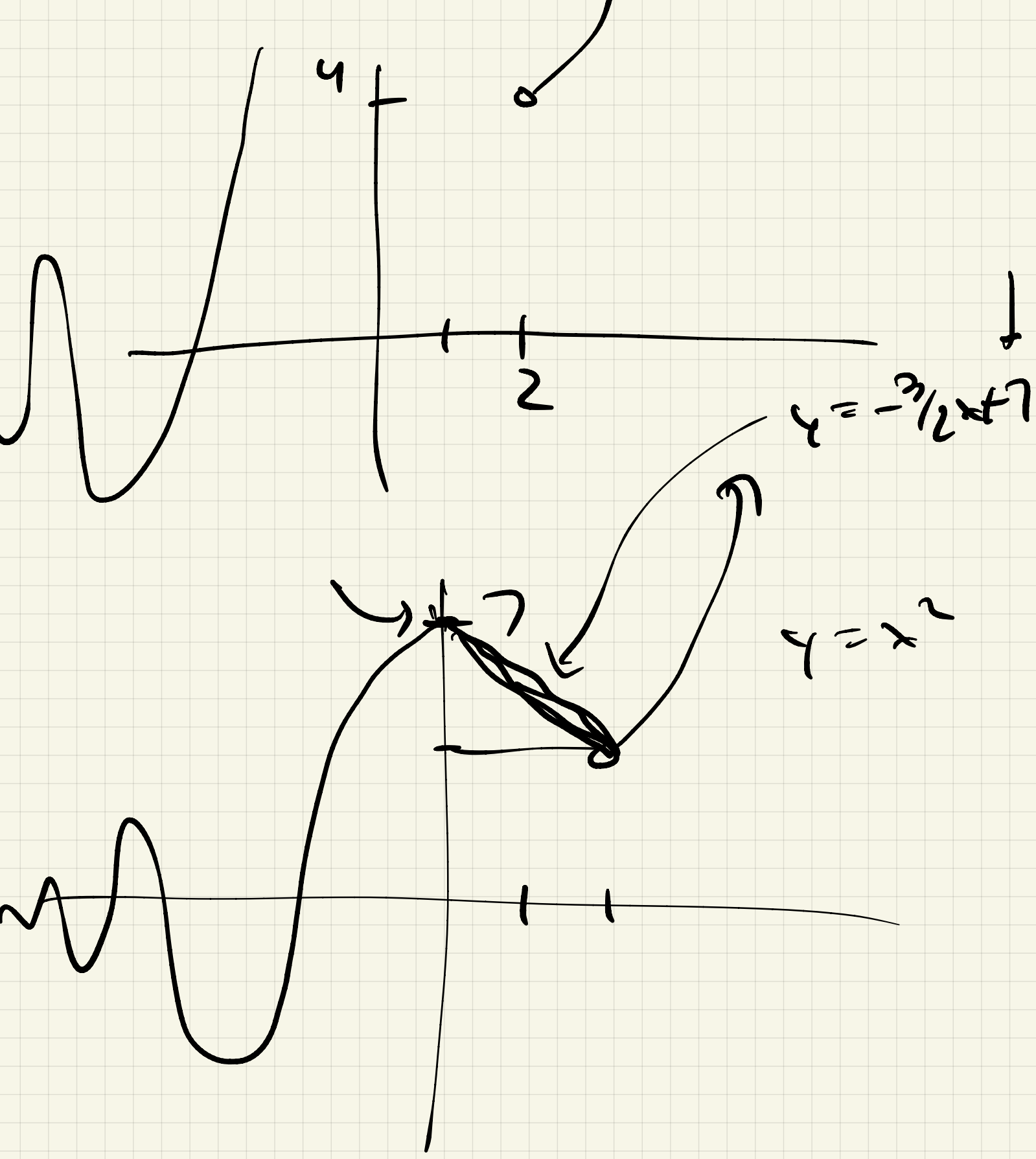
$\boxed{b=7}$

$2a+7=4 \Rightarrow$

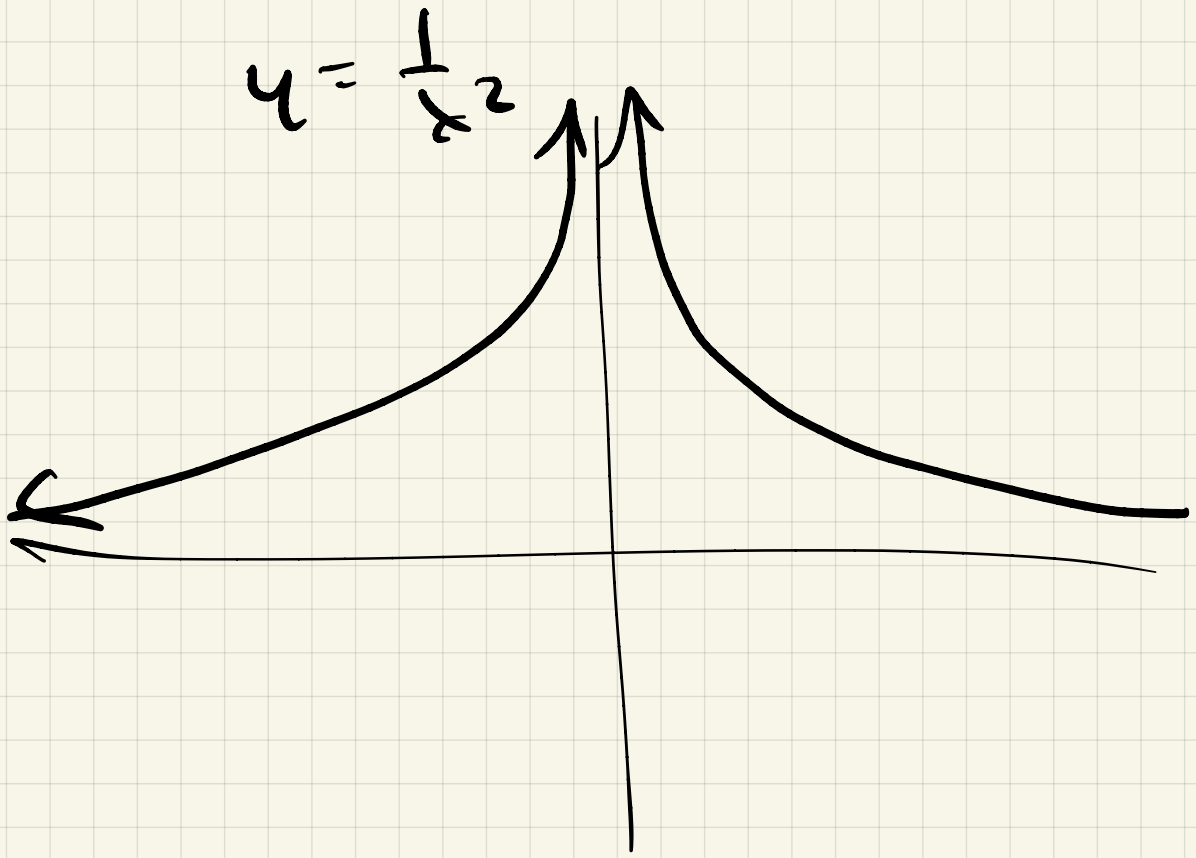
$2a=-3$

$a=-3/2$

$a=-3/2, \ b=7$



2.6 Limits Involving Infinity


$y = \frac{1}{x^2}$